

DIRECT DETECTION OF DARK MATTER

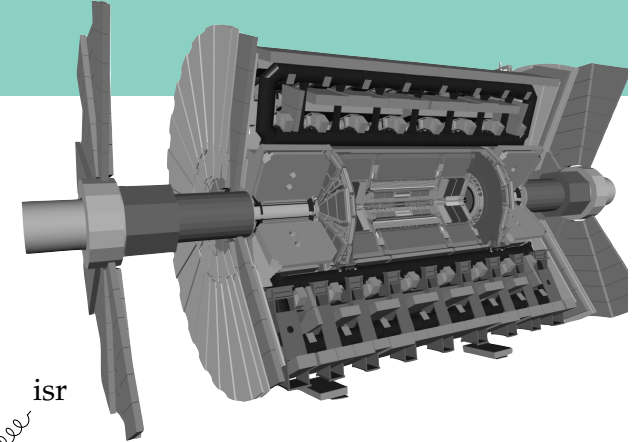
DIRECT DETECTION OF DARK MATTER

Introduction

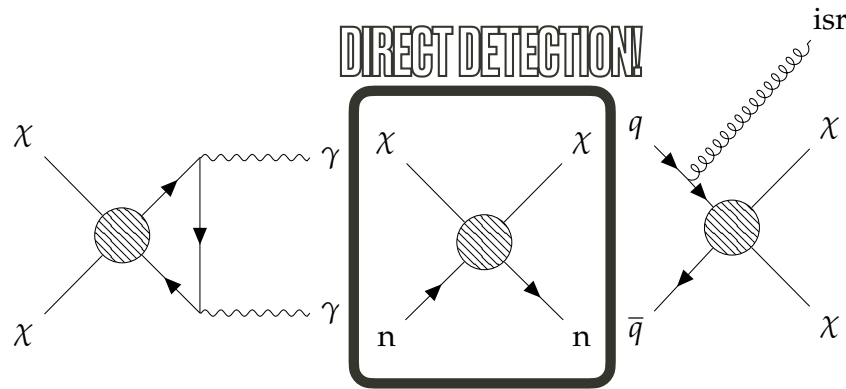
INDIRECT



COLLIDER



DIRECT DETECTION!



OVERVIEW

- **What is direct detection?**
 - **Kinematics, model, general introduction**
- **Interpreting rare event searches**
 - **Low-number statistics, reporting conventions**
- **Liquid xenon detectors**
- **Low-threshold searches**
 - **Sensors, background mitigations**
- **Other approaches and upcoming detectors**
- **Plus hands-on (or at least hands-on-computer) exercises**

There are references throughout— but if you see a topic that interests you and wish to read more, feel free to reach out!

General suggestion: always check if the Particle Data Group reviews have something to say!
https://pdg.lbl.gov/2026/reviews/contents_sports.html

Hands-on exercises at:
https://github.com/kdund/direct_detection_ictp_2026

SOME NICE REVIEWS IF YOU'RE INTERESTED

G. Bertone and D. Hooper. “A History of Dark Matter” Rev. Mod. Phys. 90, 45002 (2018) arXiv:1605.04909

-readable, and covers a lot of interesting history of the field

D. S. Akerib et al. Snowmass2021 Cosmic Frontier Dark Matter Direct Detection to the Neutrino Fog. In 2022 Snowmass Summer Study, 3 2022.

- Strategy and planning for the field

T. Marrodán Undagoitia & L. Rauch “Dark matter direct-detection experiments” J. Phys. G43 (2016) no.1, 013001 arXiv:1509.08767

-Good overview of standard direct detection searches

LECTURE PLAN

This lecture will give an introduction to direct detection, the energy scales, quick introductions to many topics

The second will focus on statistical interpretation of rare event searches, preparing you for the hands-on exercise

In the first hands-on exercise you'll try your hand at manipulating a likelihood very close to what xenon experiments use, and think about how rare event searches work

Then we'll do a deep dive into the largest and most prominent detectors; two-phase liquid xenon TPCs

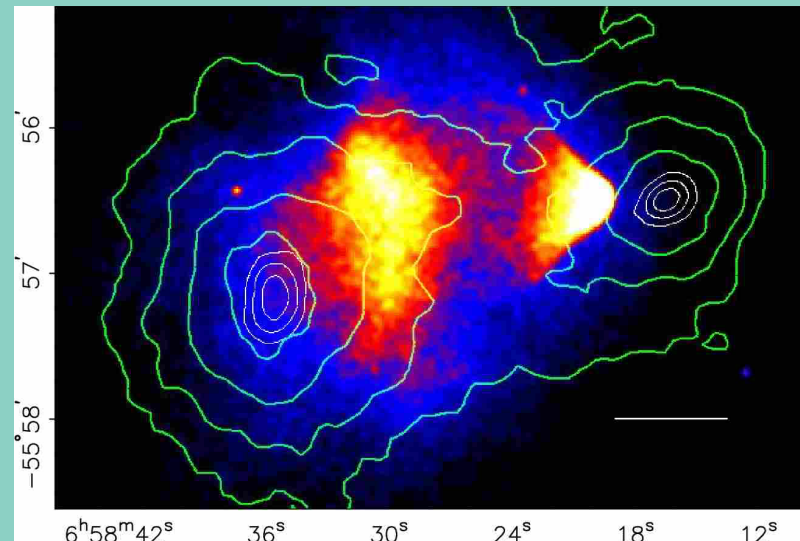
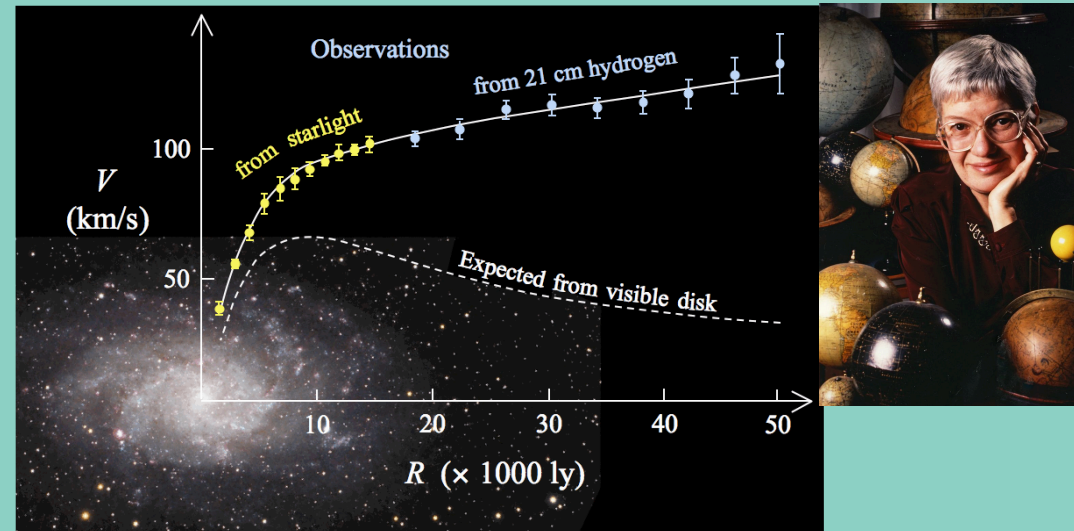
Before discussing the drive to develop extremely low-threshold experiments

Before finishing with diverse; annual modulation, a more extensive look at direct detection, and argon detectors

If time permits, you'll also give fitting simulated low-threshold data here, to get a feel for how different it looks :)

DARK MATTER

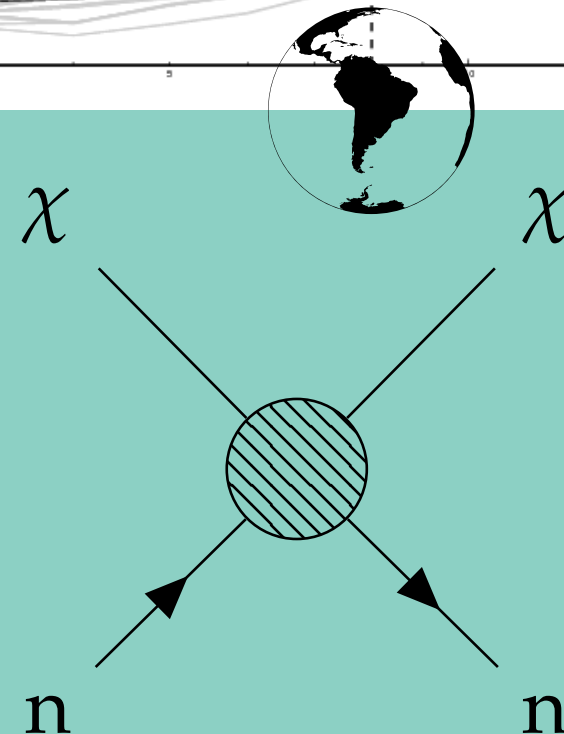
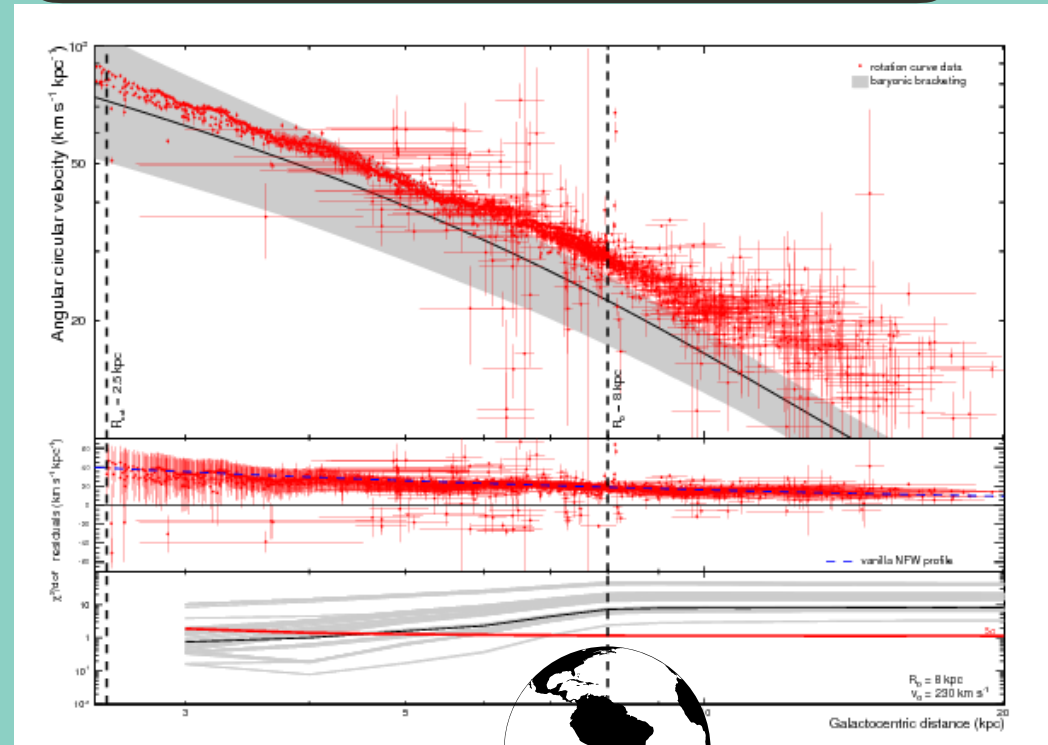
- Cosmological and dynamical evidence are all consistent with ordinary matter being only a small part (15% or so) of matter in the universe
- Measurements of stars and gas-clouds orbiting in galaxies (and mini-galaxies) indicate that galaxies are large dark matter haloes with ordinary matter collected together in the center
- Hot gas in galaxy clusters emit X-rays consistent with a similarly higher gravitational potential
- Cluster mergers/collisions show that the mass distributions passes through without interacting
- Large-scale structures in the universe of which galaxies are a part also show evidence of being held/pulled together by dark matter
- Lastly, very precise measurements of the cosmic microwave background is consistent with visible matter in the early universe having moved/oscillated alongside with a non-interacting gravitating fluid of the same abundance



WHY DIRECT DETECTION

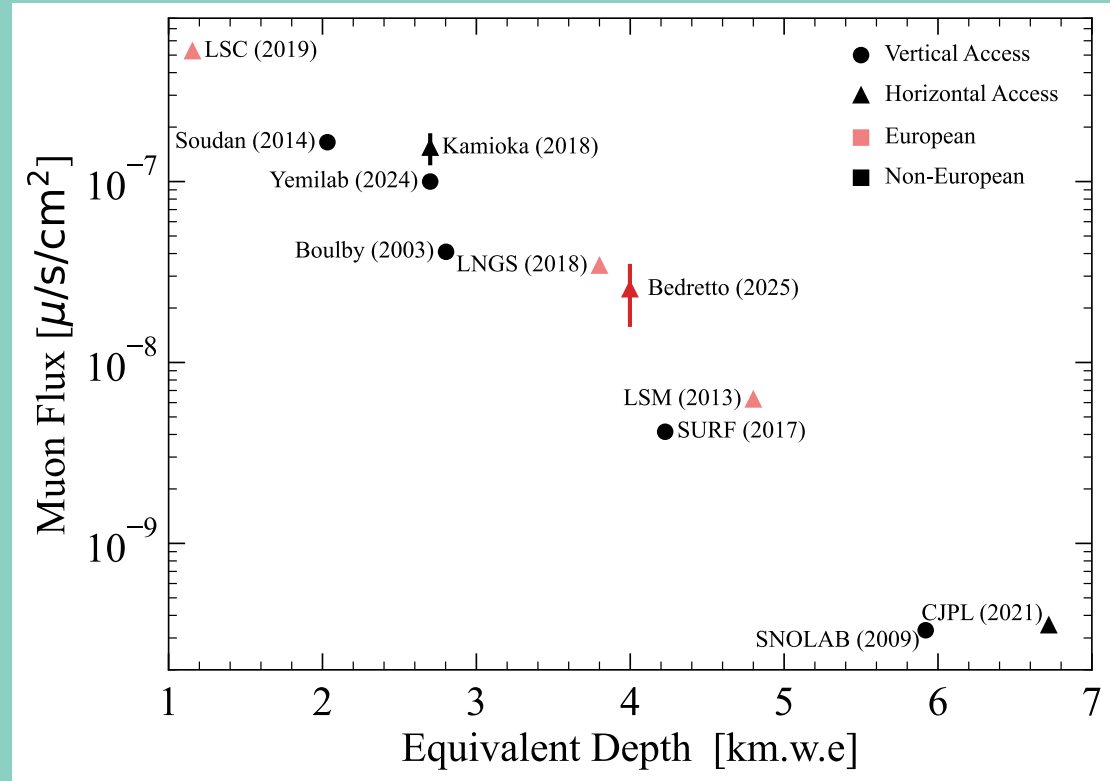
- We currently observe dark matter through its gravitational interactions
- The rotation of galaxies indicate that a dark matter “halo” extends out to much higher radii than the visible stars, and makes up most of the total mass
- Therefore, dark matter is *right here with us!*
- Direct detection aims to build detectors sensitive to dark matter interacting with an instrumented target
 - Scattering?
 - Absorbtion?
 - Excitation?

Dark matter interacting in some unknown way with a nucleus “n”



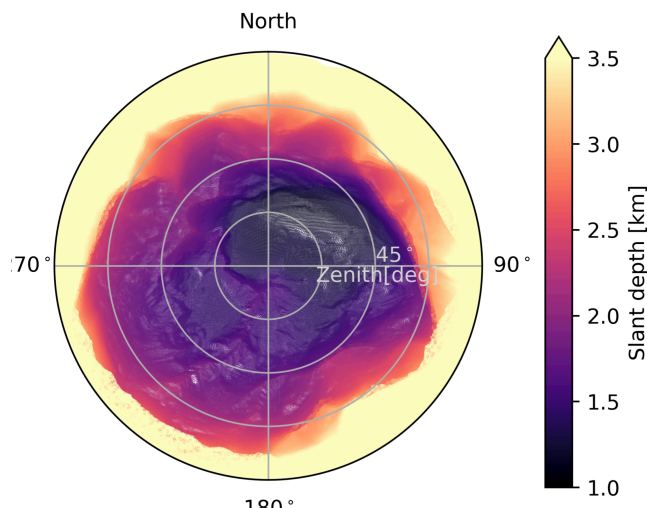
UNDERGROUND LABORATORIES

- While dark matter hardly interacts at all with ordinary matter, most backgrounds from either radioactivity or cosmic rays does interact, and can be shielded with O(m) of lead, water etc.
- Exceptions include ν (impossible to shield against) and μ ons. It is in particular to escape from muons that most dark matter searches are located in underground laboratories
- Additional benefits include more stable temperature and vibration environment, and lower activation of detector materials that would otherwise become slightly radioactive over time from cosmic ray interactions.



UNDERGROUND LABORATORIES

- Underground labs have widely varying ease of access
 - some you can drive a truck into (LNGS, Jinping)
 - Some require a mine shaft (SURF)
- Radon levels, rock overburden (composition and shape) and competing mining operations influence work
- Space always at a premium

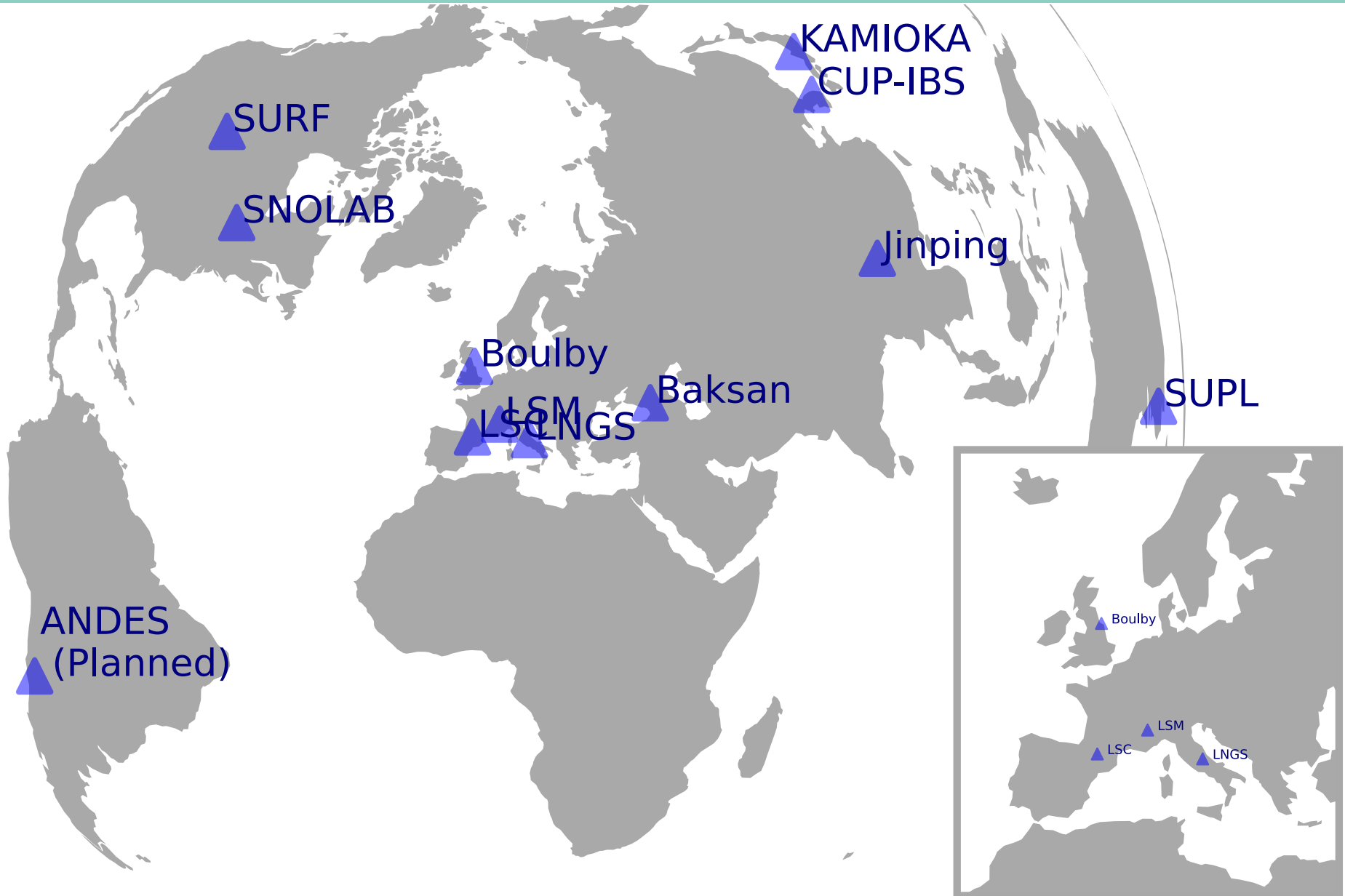


<https://news.fnal.gov/2019/10/dark-matter-experiments-central-component-takes-a-deep-dive-nearly-a-mile-underground/>

Characterisation of the Bedretto Underground Site for Fundamental Physics Experiments
, 2512.14815

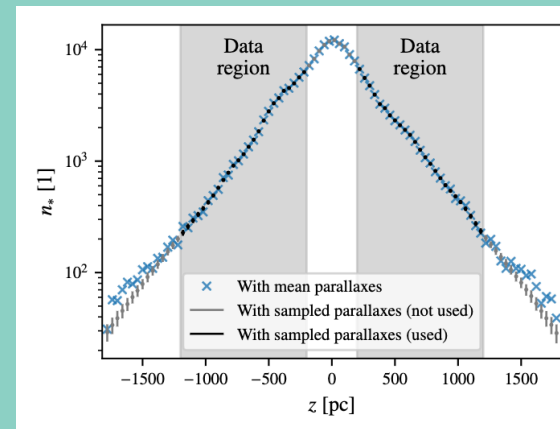


LOCATION OF UNDERGROUND LABS

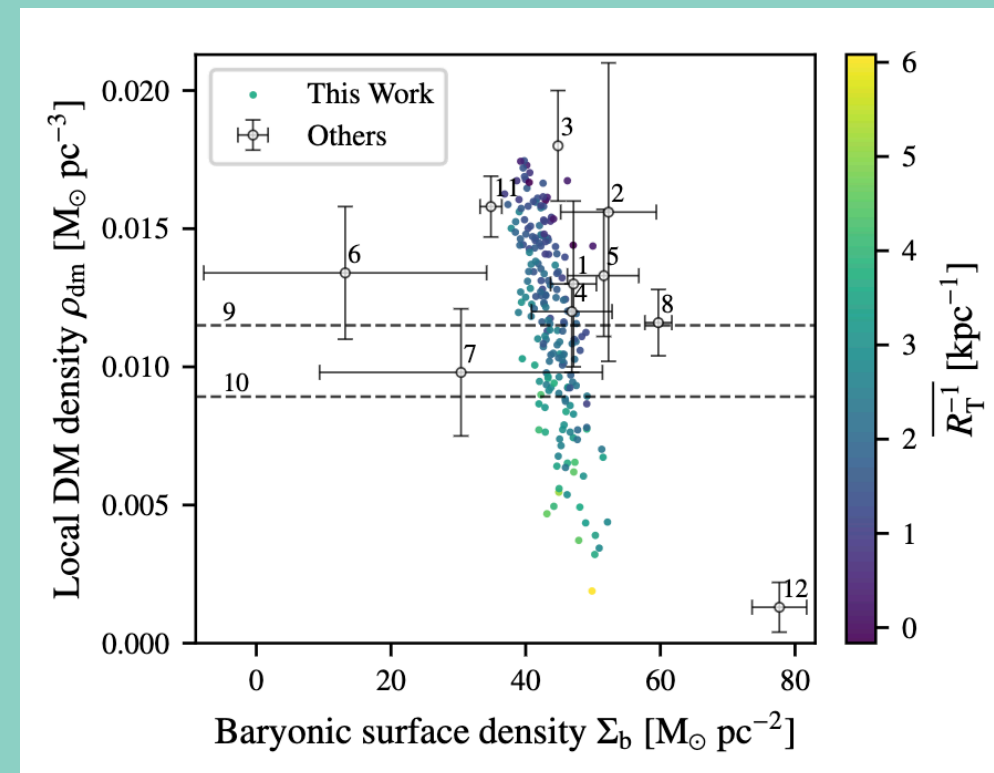


DETERMINING THE LOCAL DARK MATTER DENSITY

- The movement of stars in the Milky Way depends on the total gravitational potential, including both matter (concentrated in a disk) and the dark matter halo
- In particular the movement of stars “up and down” into the galactic disk depends on the dark matter-to-matter distribution.
- Treating a well-measured population of stars as tracers and measuring v_R, v_z constrains the potential Φ
- Measuring proper motion of hundreds of thousands of stars from Gaia allows much more complex models than before, but the value is stable, ca. $0.4\text{GeV}/c^2$



$$\overline{v_z^2} \frac{\partial v}{\partial z} + v \left(\frac{\partial \Phi}{\partial z} + \frac{\partial \overline{v_z^2}}{\partial z} + \frac{1}{Rv} \frac{\partial (Rv \overline{v_z v_R})}{\partial R} \right) = 0.$$



Local dark matter density from Gaia DR3 K-dwarfs using Gaussian processes, 2506.02956

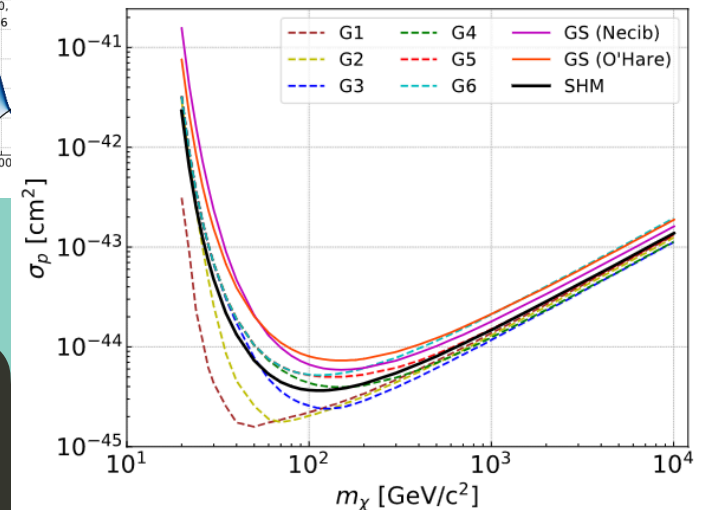
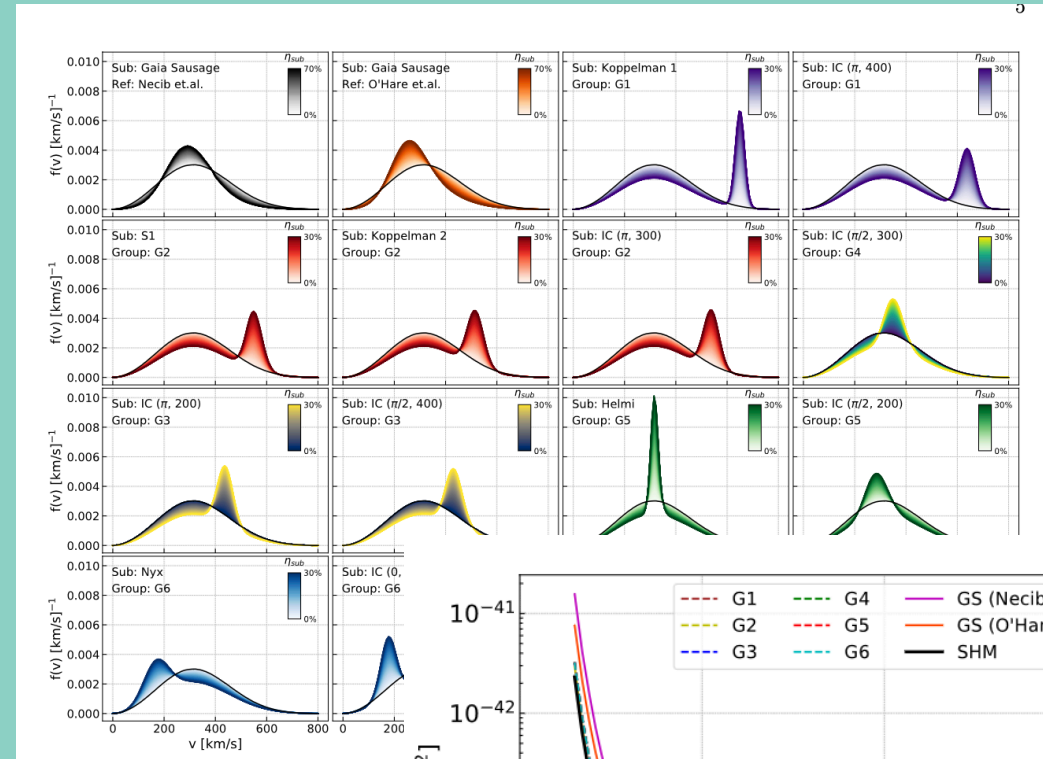
THE DARK MATTER VELOCITY DISTRIBUTION

- If dark matter scatters elastically in our detector, the only available energy is the kinetic energy of the dark matter (and the Earth) orbiting the Galaxy
- The energy spectrum of a dark matter signal depends on this distribution $f(v)$
- As does the total expected rate if the detector threshold is not much lower than the available kinetic energy
 - in particular liquid xenon detectors for low-mass
- Gaia data has shown that there are substructure in the stellar population in phase space, a boosted sub-population or a dark disk can boost the energy or number of dark matter particles
- For now, results are almost always reported with the standard halo model

$$\vec{v}_{\text{gal}} = \vec{v}_{\text{lab}} + (\vec{v}_0 + \vec{v}_{\otimes} + \vec{v}_{\oplus}(t))$$

$$f_{\text{MB}}(\vec{v}) \propto \begin{cases} e^{-|\vec{v}|^2/v_0^2} & |\vec{v}| < v_{\text{esc}} \\ 0 & |\vec{v}| \geq v_{\text{esc}}. \end{cases}$$

“Standard Halo Model” from Recommended conventions for reporting results from direct dark matter searches, 2105.00599v3

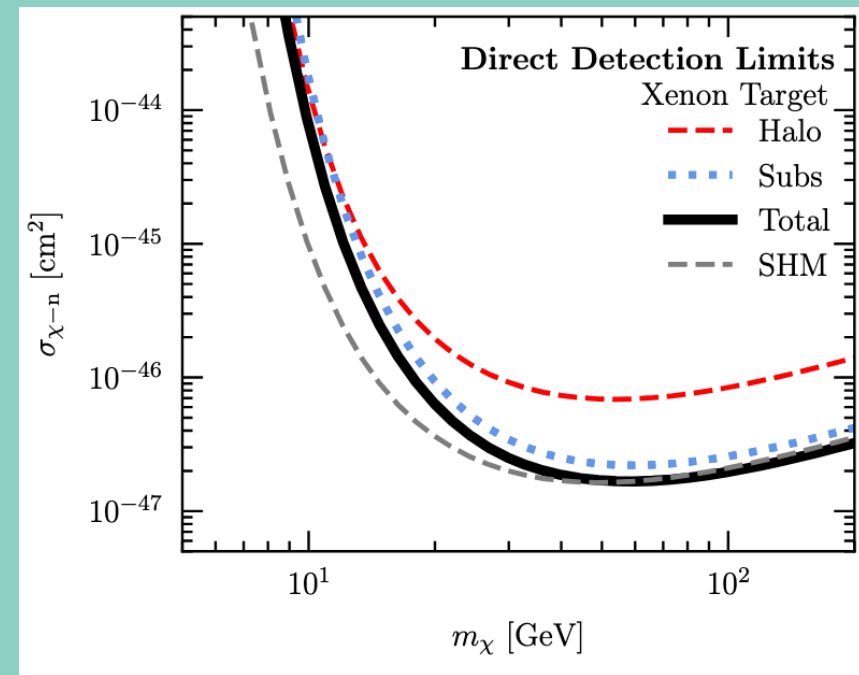
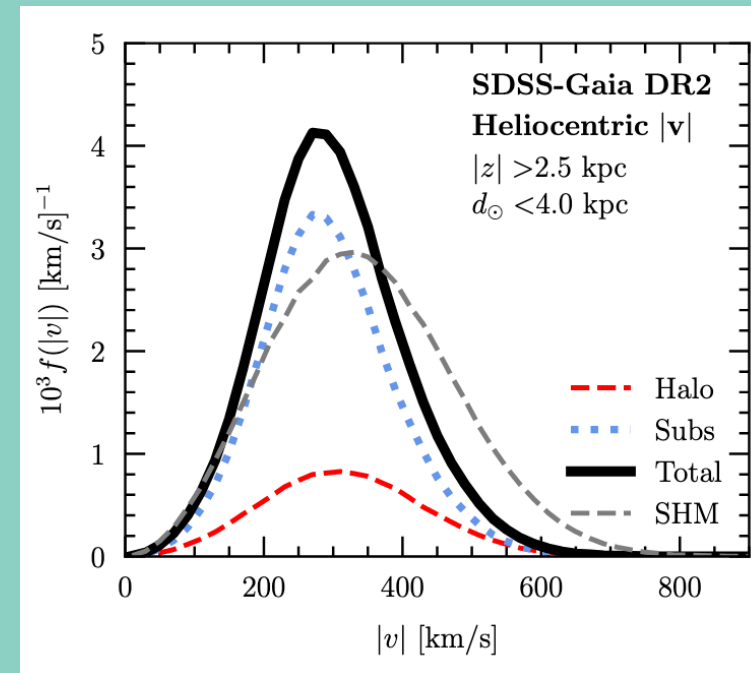


(a) \mathcal{O}_1 interactions

Constraints on dark matter-nucleon effective couplings in the presence of kinematically distinct halo substructures using the DEAP-3600 detector, 2005.14667

THE DARK MATTER VELOCITY DISTRIBUTION

- Substructures may also make limits looser, if it turns out that a significant fraction of the dark matter near us is moving with the sun
- For example if a sub-structure of the galactic halo is found moving in roughly the same direction as the sun
- or if a portion of the dark matter is aligned with and moving with the galactic disk
- No strong evidence for a disk has been found, and assumptions about the Milky Way accretion history makes any strong statements about subhalos difficult
- Primarily important to keep in mind that the standard halo model is an assumption we make



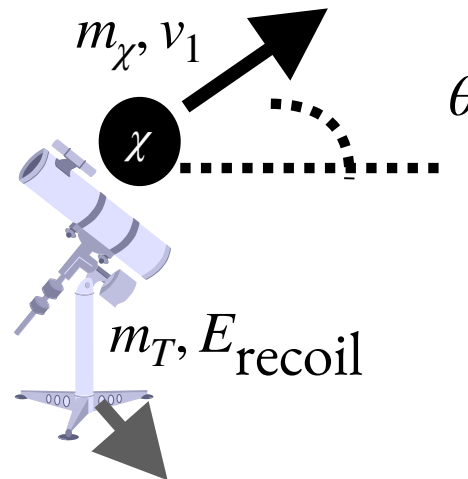
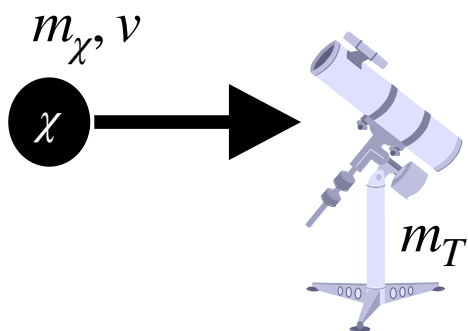
DARK MATTER- MATTER SCATTERING

Review of mathematics, numerical factors, and corrections for dark matter experiments based on elastic nuclear recoil, Lewin and Smith Astroparticle Physics 6,1 87-112 (1996)

- Assuming dark matter may interact weakly with ordinary matter, it can deposit energy in elastic scatters where it bounces off a standard model particle in our target, depositing a recoil energy
- Considering energy and momentum conservation gives :
- Which gives a maximal deposited energy of

$$E_{\text{recoil}} = \frac{\mu^2 v^2}{m_T} (1 - \cos \theta)$$
$$\mu \equiv \frac{m_\chi^2 m_T^2}{(m_\chi^2 + m_T^2)}$$

$$E_{\text{recoil}}^{\text{max}}(v) = \frac{2\mu^2 v^2}{m_T}$$



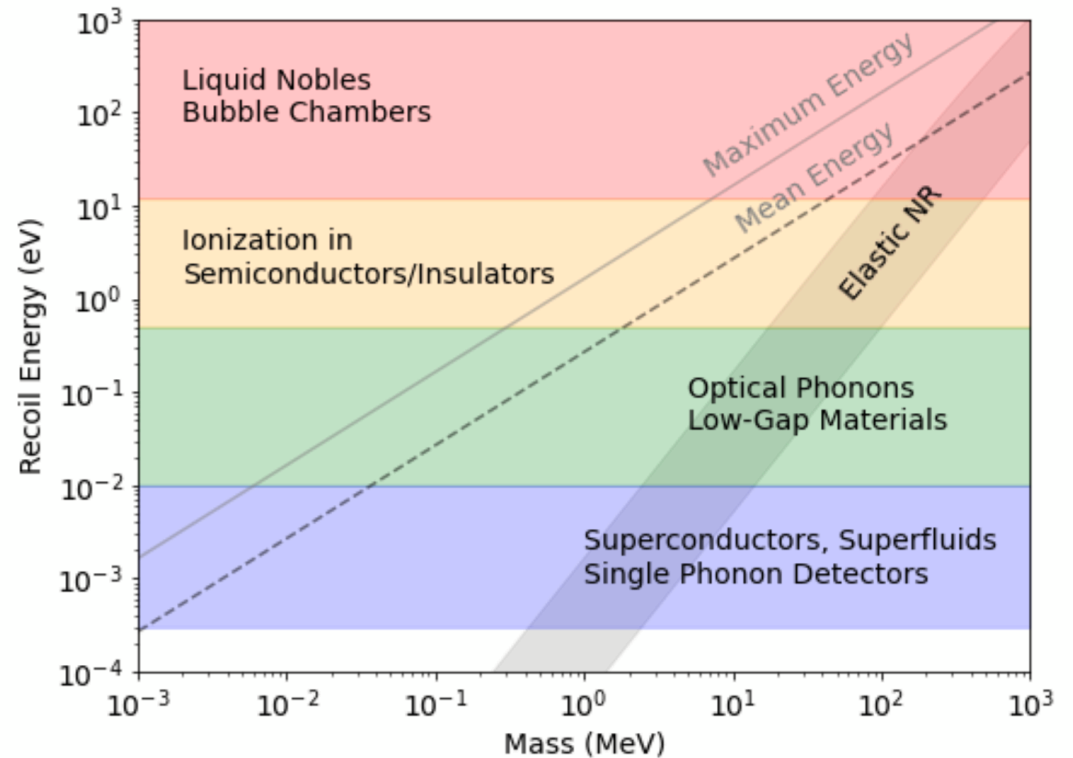
DARK MATTER- MATTER SCATTERING

- The choice of target therefore affects the energy you can get out, which along with energy threshold drives detector target choices. The highest energy is when $m_T = m_\chi$

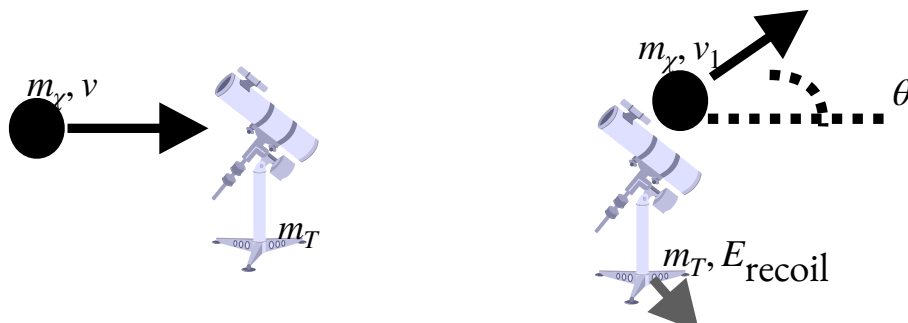
$$m_T = m_\chi \implies E_{\text{recoil}}^{\text{max}}(v) = \frac{m_\chi}{2} v^2$$

$$m_T \ll m_\chi \implies E_{\text{recoil}}^{\text{max}}(v) = 2m_T v^2$$

$$m_\chi \ll m_T \implies E_{\text{recoil}}^{\text{max}}(v) = 2m_\chi^2 v^2 / m_T$$



Snowmass2021 Cosmic Frontier:
The landscape of low-threshold dark matter
direct detection in the next decade
2203.08297v2

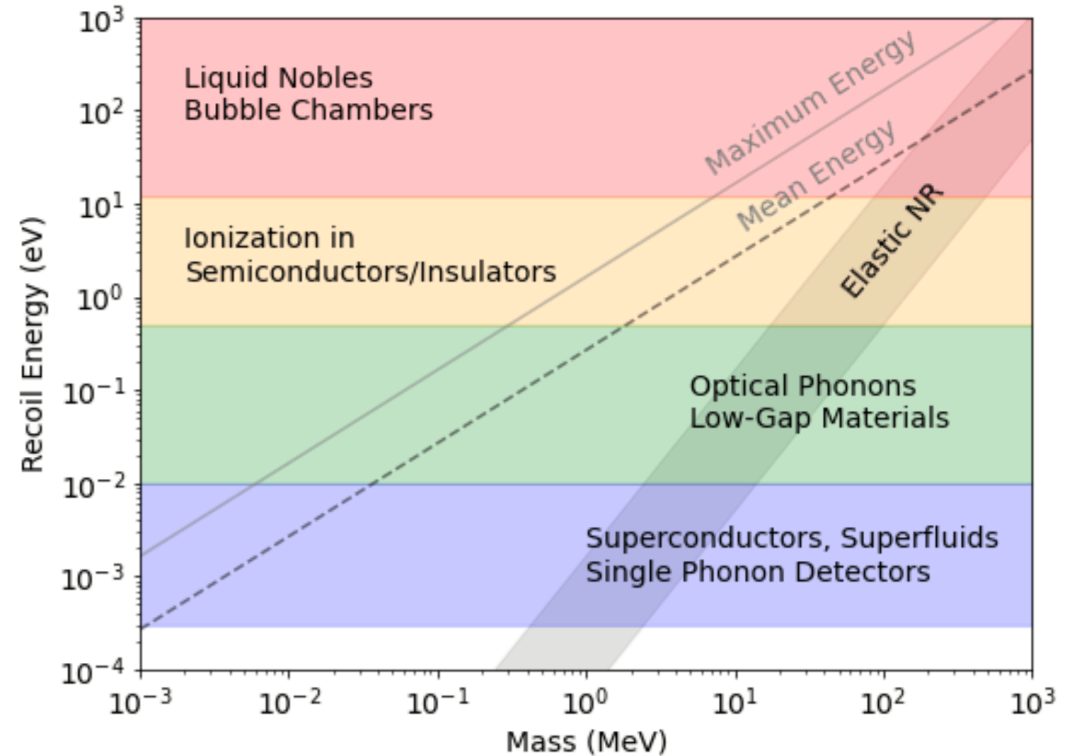


DARK MATTER- MATTER SCATTERING

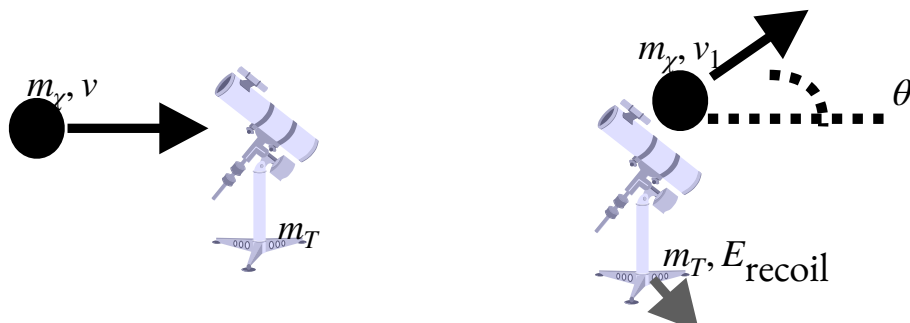
$$v \sim 220 \text{ km/s} = 7 \times 10^{-4} c$$

$$m_T = m_\chi \implies E_{\text{recoil}}^{\text{max}}(v) = \frac{m_\chi}{2} v^2$$

- Maximum deposited energy for a well-matched (i.e. $m_T = m_\chi$) target for:
 - Weak-scale dark matter ($m_\chi \sim 100 \text{ GeV}/c^2$): 24keV
 - “Light dark matter” $m_\chi \sim 1 \text{ GeV}/c^2$: 2.4keV
 - $m_\chi = m_e = 0.5 \text{ MeV}/c^2$: 0.12eV

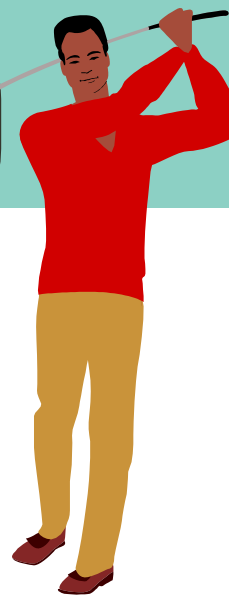


Snowmass2021 Cosmic Frontier:
The landscape of low-threshold dark matter
direct detection in the next decade
2203.08297v2



DARK MATTER- MATTER SCATTERING

Review of mathematics, numerical factors, and corrections for dark matter experiment based on elastic nuclear recoil, Lewin and Smith Astroparticle Physics 6,1 87-112 (1996)



- Within the energy range available we can integrate the dark matter velocity distribution times the dark matter-matter cross section (giving the probability of interaction) to get the total rate in the detector per detector mass :

$$\frac{dR}{dE_{\text{recoil}}} = \frac{1}{m_T} \int d^3\vec{v} \cdot \frac{\rho_0}{m_\chi} f(v) |\vec{v}| \cdot \frac{d\sigma}{dE_{\text{recoil}}}(v)$$

Number of
target particles

Flux of dark
matter particles

Probability of
interacting to
give this recoil
energy

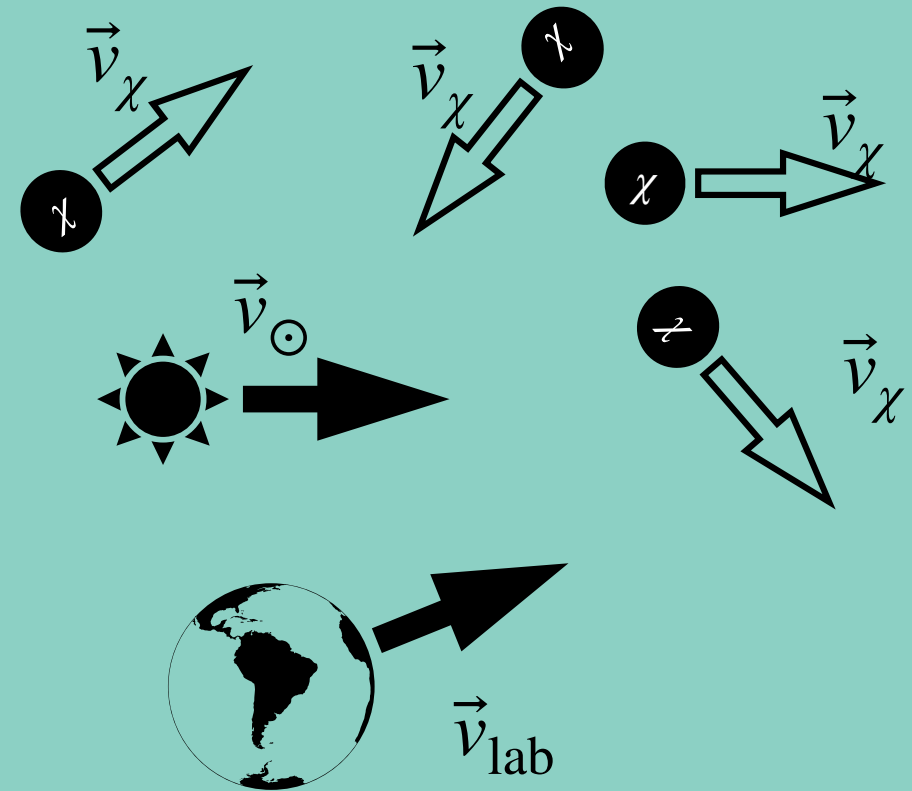
DARK MATTER VELOCITY TERM

- Recalling the simplified standard halo model, where $v_0 = \sqrt{2k_B T/m}$ is the most probable.
- The sun has a “peculiar velocity” — it is not at rest in the galactic frame, but orbits with $v_\odot \sim 232\text{km/s}$ roughly along the galactic plane.
- For now we’ll assume that the Earths orbit with $v_{\text{Earth}} \sim 30\text{km/s}$ averages out
- Unless the dark matter-matter cross-section has a strong dependence on deposited energy this term will dominate the expected recoil energy distribution

$$\vec{v}_{\text{gal}} = \vec{v}_{\text{lab}} + (\vec{v}_0 + \vec{v}_\otimes + \vec{v}_\oplus(t))$$

$$f_{\text{MB}}(\vec{v}) \propto \begin{cases} e^{-|\vec{v}|^2/v_0^2} & |\vec{v}| < v_{\text{esc}} \\ 0 & |\vec{v}| \geq v_{\text{esc}}. \end{cases}$$

“Standard Halo Model” from Recommended conventions for reporting results from direct dark matter searches, 2105.00599v3



DARK MATTER- MATTER CROSS SECTION

- We express the interaction strength between dark and ordinary matter with a cross-section, proportional to the probability of interacting
- We expect the transferred recoil energy to be small, and so for many models, the interaction can be split into a constant cross-section term and a form factor F that encodes the dark matter-nucleus scattering energy dependence
 - $F(0) = 1$ by convention
 - for intuition, $F = 1$ corresponds to a hard sphere
- These collisions can be considered non-relativistic when the transferred momentum q is significantly smaller than the particle masses

$$\frac{d\sigma}{dE_{\text{recoil}}}(v) \equiv \sigma_0 F^2(E_{\text{recoil}}) / E_{\text{max}}(v)$$

0-momentum
scattering
cross-section

Form factor



$$q = \sqrt{2E_{\text{recoil}}m_T} < m_T, m_\chi?$$

DARK MATTER- MATTER CROSS SECTION

- For an illustrative example, we'll consider a very heavy weakly interacting particle colliding on a xenon atom. In this case, the maximum deposited energy (rarely reached) means the transferred momentum is far below the target mass.
- The scattering may therefore be considered entirely nonrelativistic, with cross-sections depending only on momentum transfers, spins and the dark matter velocity.
- The least suppressed interaction is the “spin-independent” interaction since it couples to all nucleons rather than only the one unpaired spin and does not include any suppression by q or v

$$v_{\text{esc}} + v_{\odot} = 544 + 220 \text{ km/s} = 2.5 \times 10^{-3}$$

$$E_{\text{recoil}}^{\text{max}}(v_{\text{esc}}) = 2m_T v_{\text{esc}}^2,$$

$$E_{\text{recoil}}^{\text{max}}(v_{\text{esc}}) = 1.7 \text{ MeV}$$

$$q = \sqrt{2E_{\text{recoil}} m_T} = 0.76 \text{ GeV} \lll 131 \text{ GeV}$$

$$\frac{d\sigma}{dE_{\text{recoil}}}(v) \approx \frac{1}{E_{\text{max}}(v)} \left(\sigma_0^{\text{SI}} F_{\text{SI}}^2(E_{\text{recoil}}) + \sigma_0^{\text{SD}} F_{\text{SD}}^2(E_{\text{recoil}}) \right)$$

SPIN-INDEPENDENT NUCLEAR FORM- FACTOR

- Considering only the spin-independent form-factor
- The Helm form factor is analytic, and close enough to experimental data to be used for spin-independent scattering
- At typical xenon recoil energies, F is on the order 1, while the suppression reaches 10^{-8} for the highest possible xenon recoil energies

momentum

transfer:

$$q = \sqrt{2E_{\text{recoil}}m_T}$$

$$\hat{q} \equiv q/(\hbar c)$$

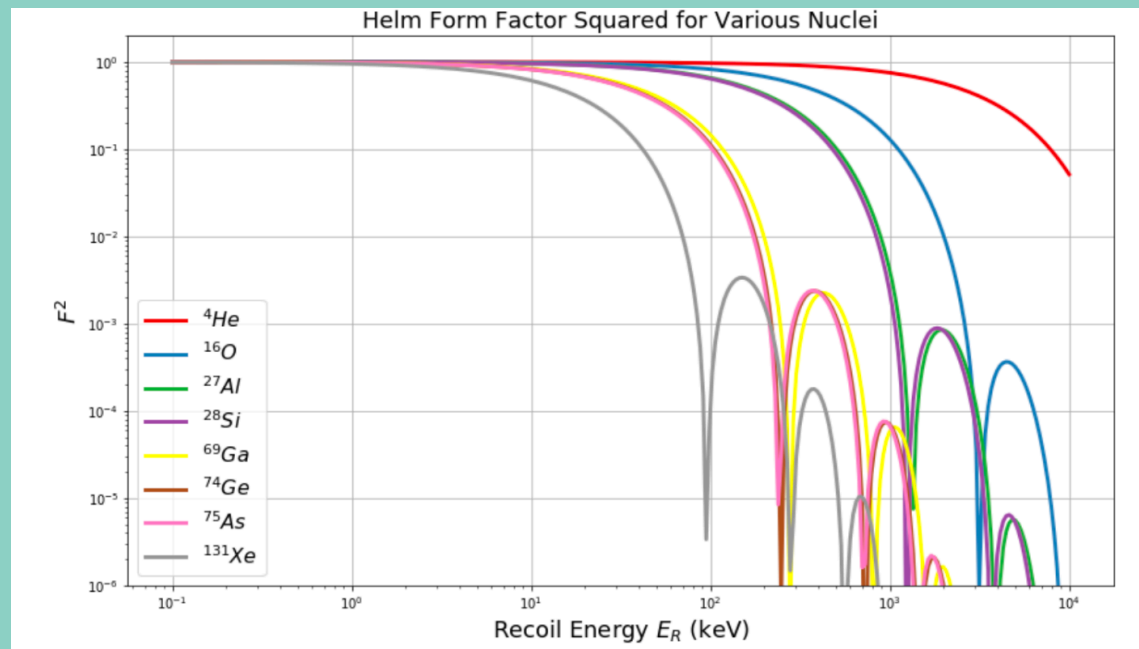
$$s = 0.9\text{fm}$$

scale factor
function is a

$$r \sim A^{1/3}\text{fm}$$

Review of mathematics, numerical factors, and corrections for dark matter experiments based on elastic nuclear recoil, Lewin and Smith Astroparticle Physics 6,1 87-112 (1996)

$$F(E) = \frac{3j_1(\hat{q}r)}{\hat{q}r} e^{-(\hat{q}s)^2/2}$$



0-MOMENTUM CROSS SECTION

- The last ingredient we need is the spin-independent zero-recoil energy cross-section, which if the DM-nucleon coupling f is equal for protons and neutrons equals
- The scattering amplitude of each nucleon adds coherently, gaining the rate the coherent enhancement A^2
- To compare different detectors using different targets, detectors report results for the dark matter-nucleon cross-section instead of the cross-section on their target

$$\sigma_0^{\text{SI}} = \frac{4\hbar^2 c^2}{\pi} \mu^2 f^2 A^2$$

$$\frac{\sigma_0^{\text{SI}}}{\sigma_0^{\text{SI DM-nucleon}}} = \frac{\mu^2}{\mu_{\text{nucleon}}^2} A^2$$

TOTAL RATE

- Returning to the equation for the scattering rate, for an actual experiment we'd integrate numerically to include effects such as the escape velocity
- However, if we just include the exponential part of the maxwell-Boltzmann, it is possible to numerically integrate
- The shape is a smooth featureless spectrum multiplied by the form factor

$$\frac{dR}{dE_{\text{recoil}}} = \frac{1}{m_T} \int d^3\vec{v} \frac{\rho_0}{n_\chi} f(v) |\vec{v}| \frac{d\sigma}{dE_{\text{recoil}}}(v)$$

Number of target particles

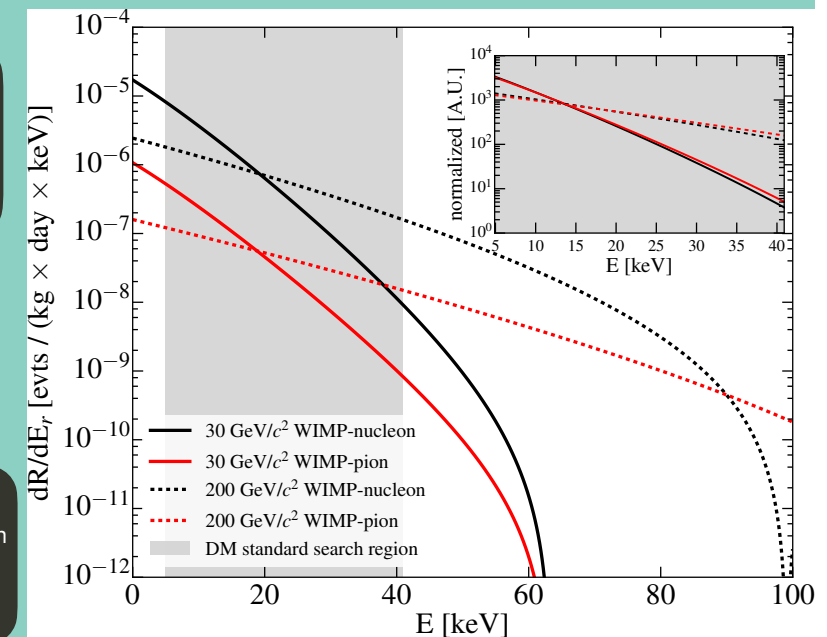
Flux of dark matter particles

Probability of interacting to give this recoil energy

$$\frac{dR}{dE_{\text{recoil}}} = \frac{R_0}{E_s} e^{E_{\text{recoil}}/E_s} \cdot F^2(E_{\text{recoil}}) \cdot A^2$$

Review of mathematics, numerical factors, and corrections for dark matter experiments based on elastic nuclear recoil, Lewin and Smith Astroparticle Physics 6,1 87-112 (1996)

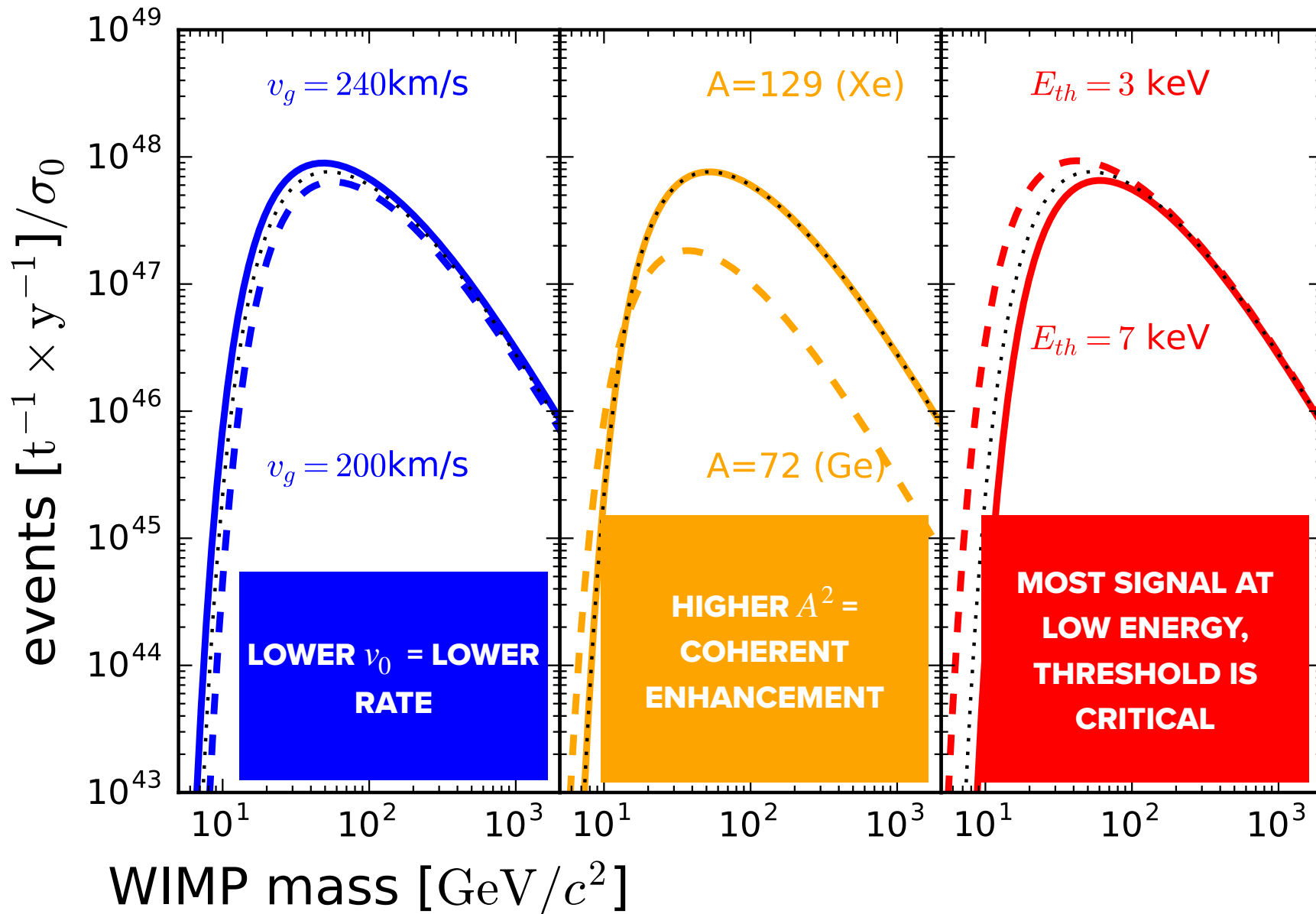
XENON collaboration. "First results on the scalar WIMP-pion coupling, using the XENON1T experiment." Physical review letters 122.7 (2019): 071301.



$$E_s = 2v_0^2 \mu^2 / m_T, R_0 = \frac{2}{\sqrt{\pi}} \frac{N_A}{A} \frac{\rho_0}{m_\chi} \sigma_0 v_0$$

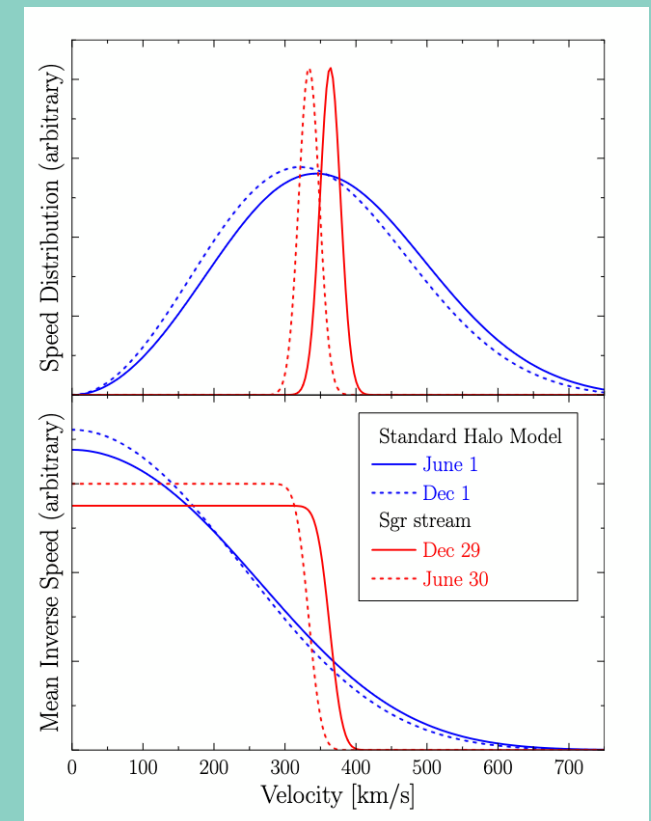
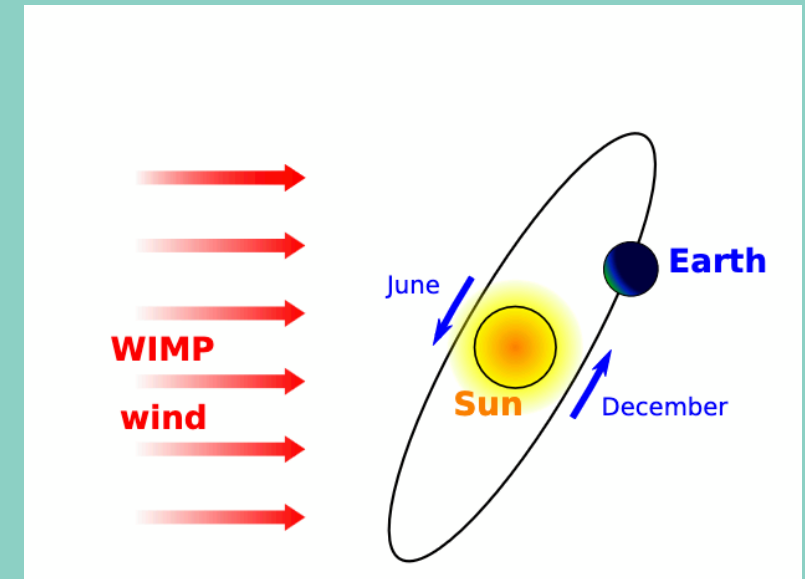
LESSONS FROM THE RECOIL SPECTRUM

$$\frac{dR}{dE_{\text{recoil}}} = \frac{R_0}{E_s} e^{E_{\text{recoil}}/E_s} \cdot F^2(E_{\text{recoil}}) \cdot A^2$$



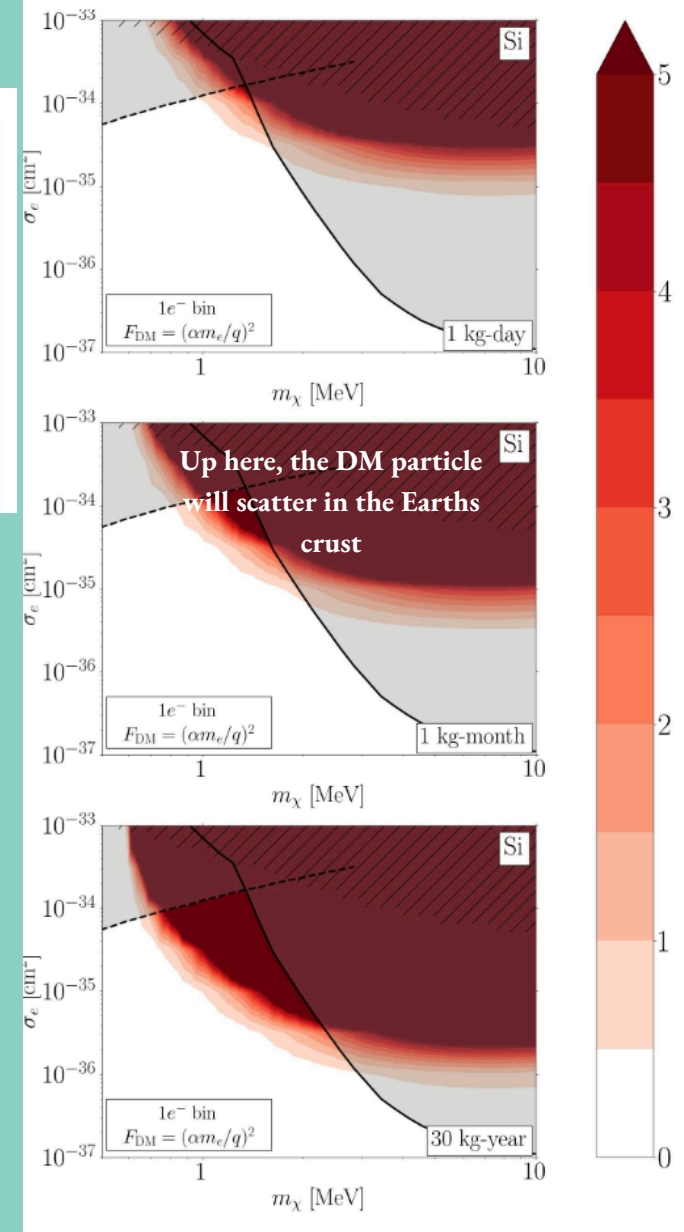
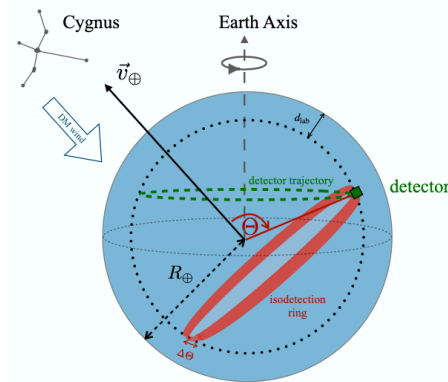
ANNUAL MODULATION

- On top of the average dark matter velocity distribution, the movement of the Earth yields a small additional energy boost when Earth is moving with the movement of the Sun, into the dark matter “wind” just around now (June)!
- Two effects: higher rate, and a higher average energy of the recoils. Depending on the energy threshold and the underlying dark matter velocity distribution, this is expected to give an amplitude of $\sim 1\%$
- While this signal will therefore only be visible in most cases when you have already seen many dark matter events, it is a crucial signature to be able to identify an excess in a direct detection experiment with dark matter.



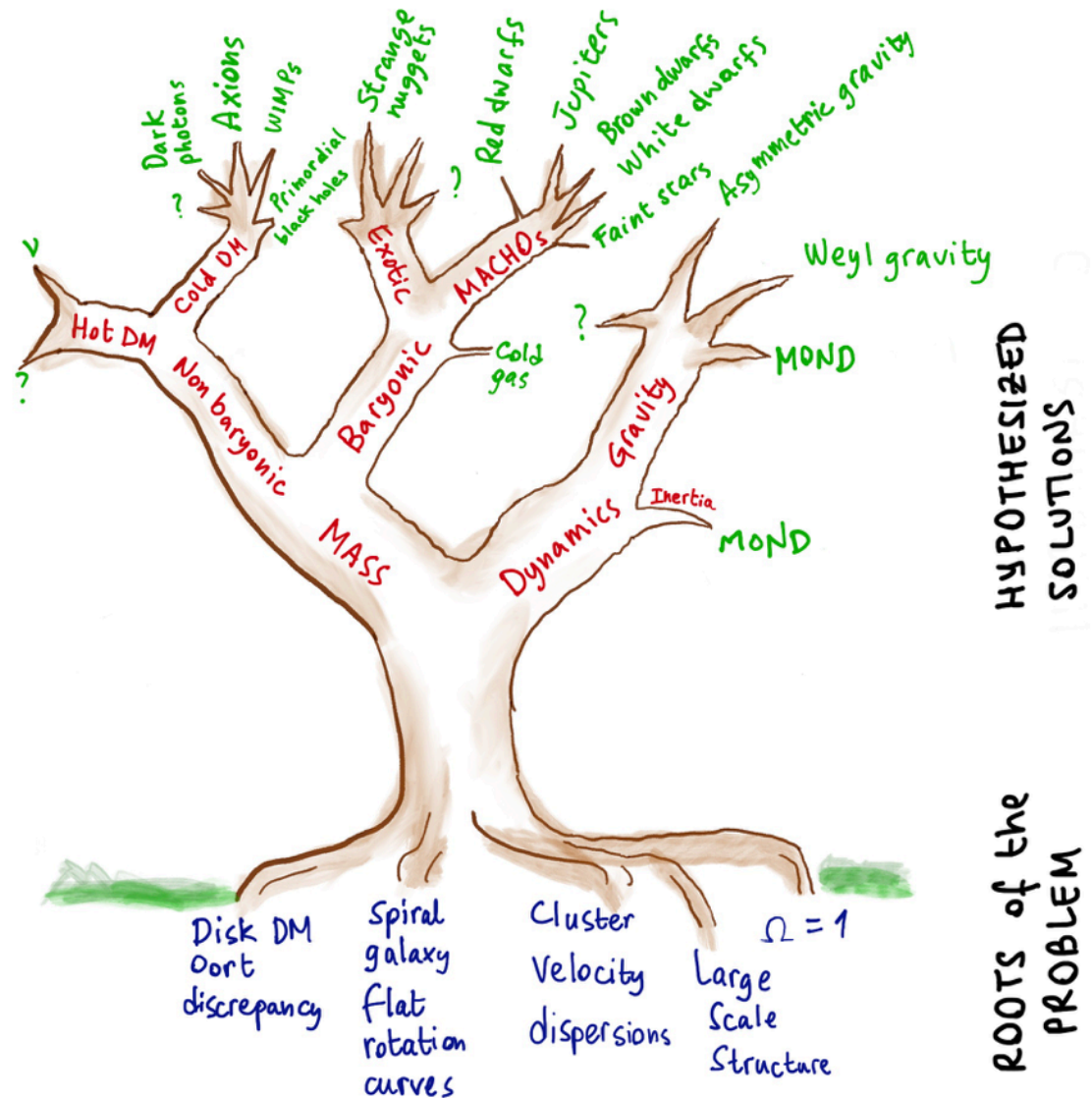
DIURNAL MODULATION

- For low-mass $< 1\text{GeV}/c^2$ dark matter, even relatively high cross-sections where dark matter interacts with the Earth's crust are not yet excluded
- In addition, the Earth and Sun's gravitational fields will slightly distort the velocity distribution
- Together, this means that some dark matter models (mainly interacting with electrons) may have a daily fluctuation as more or less of the Earth shields the detector
- For the most-scattered models, modulation rates can be as high as 50%
- Plot on right shows changing reach

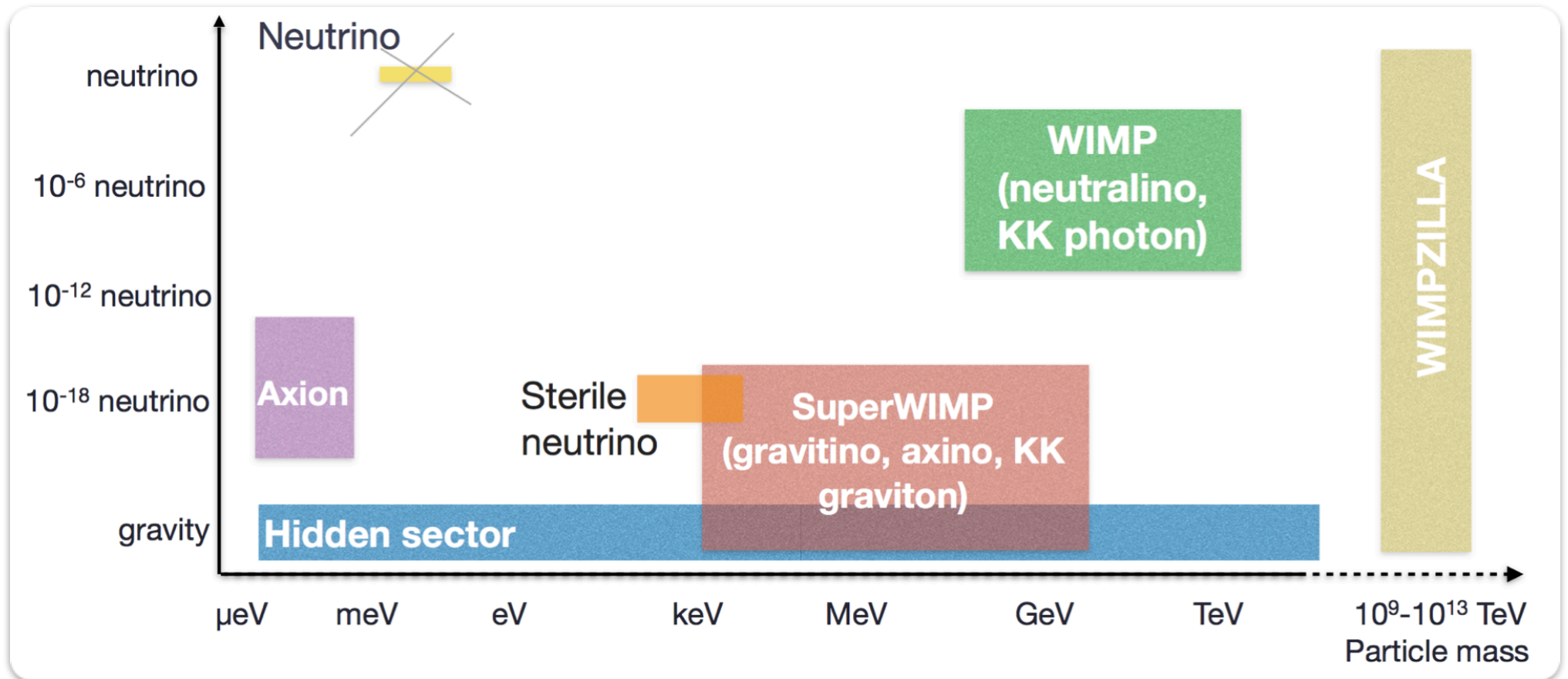


WIDE RANGE OF POSSIBLE MODELS

- No electric charge
- Massive
- Limited self-interactions (cluster mergers set limits)
- Non-relativistic in the early universe
- Stable

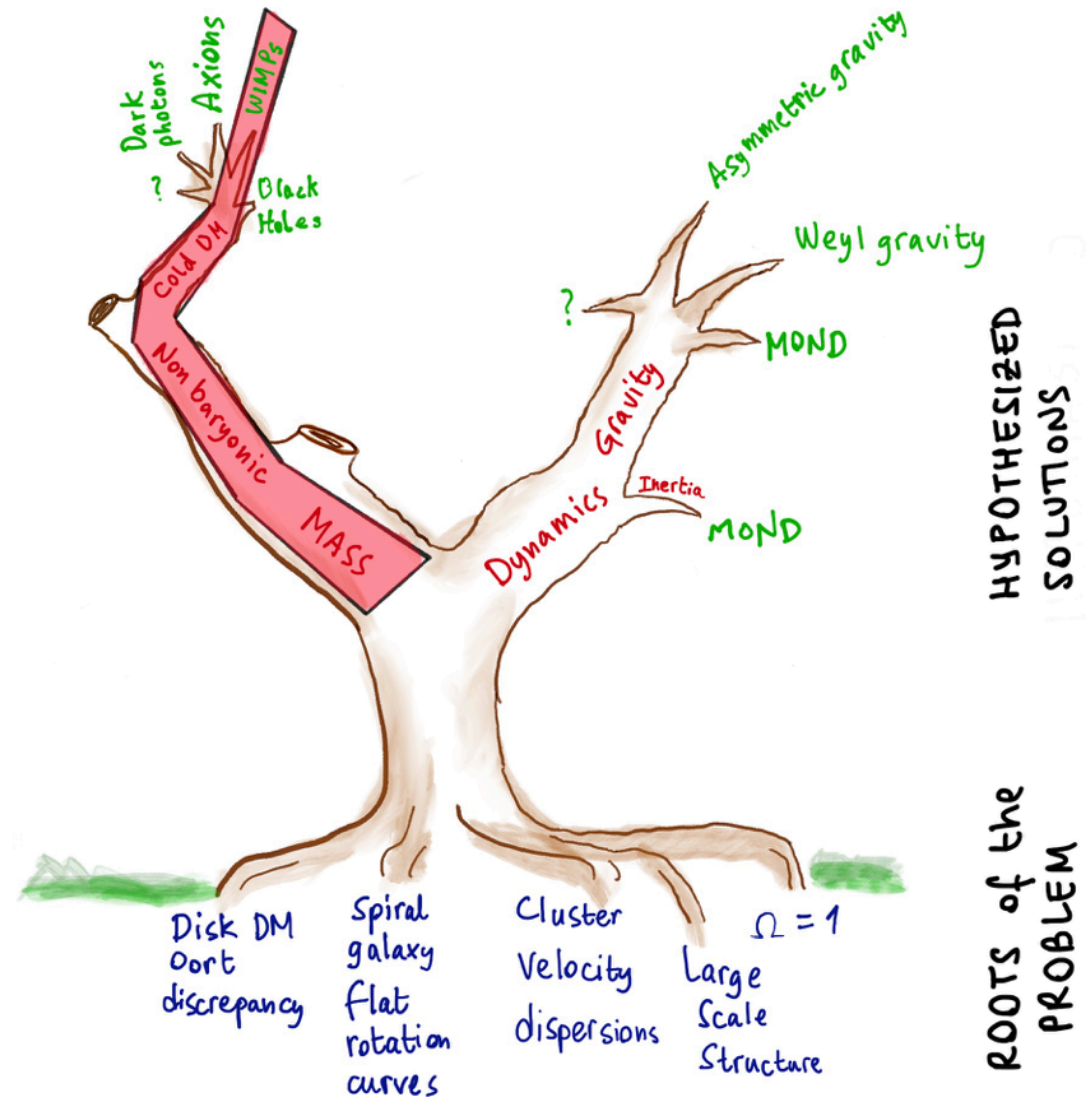


WIDE RANGE OF POSSIBLE MODELS



WIMPS: THE MODEL DARK MATTER MODEL

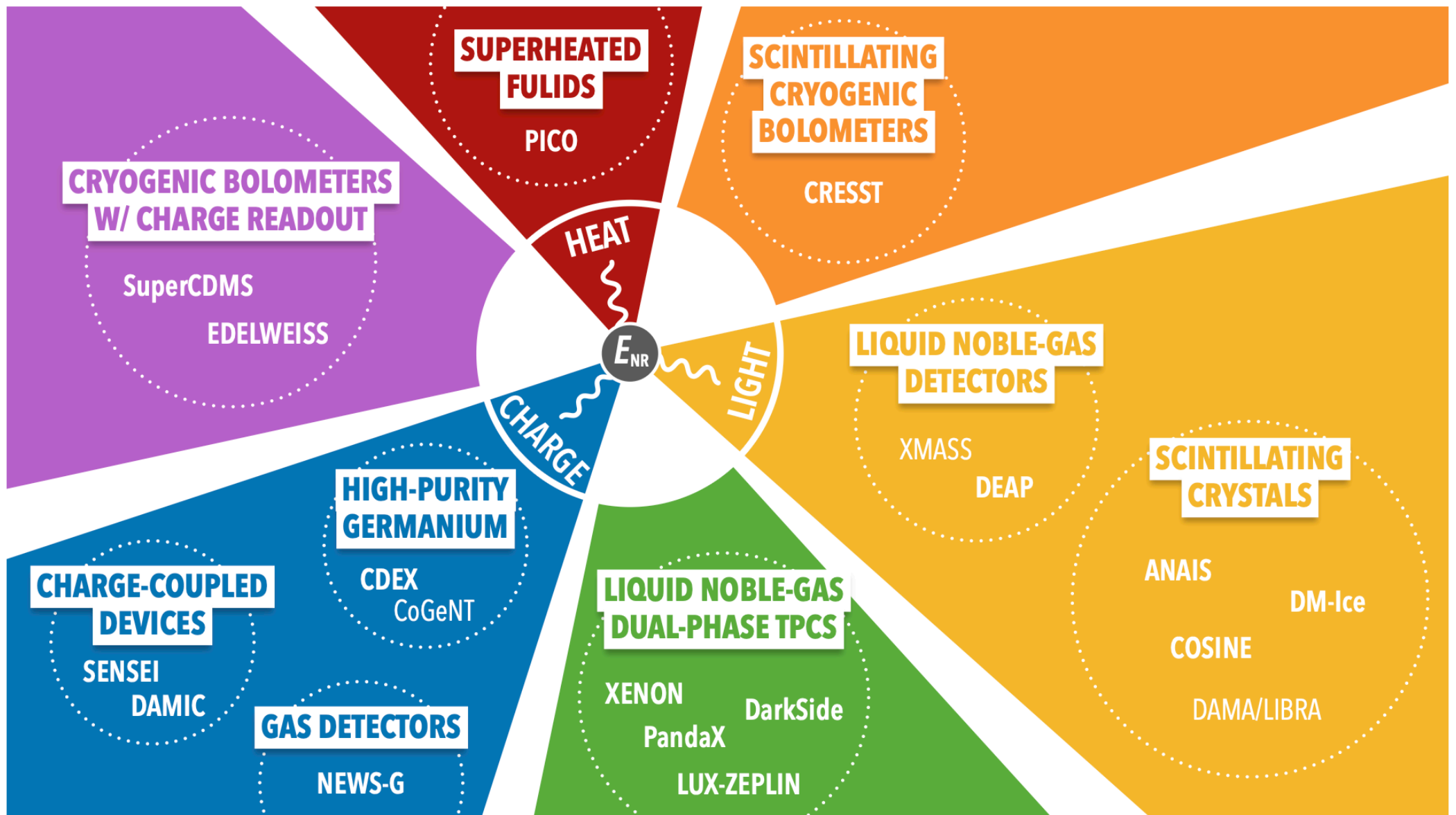
- **Weakly Interacting Massive Particles, WIMPs** remains a viable dark matter candidate, though a range of searches so far have found only null results.
- a particle with weak-scale mass ($100 \text{ GeV}/c^2$) and cross-section would be thermally produced with roughly the right relic density
- Experiments, however, typically define WIMPs as any dark matter particle with mass and weak interactions



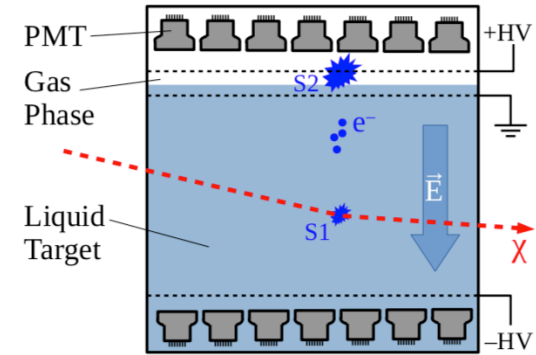
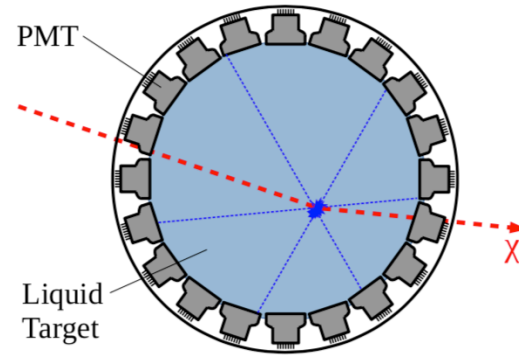
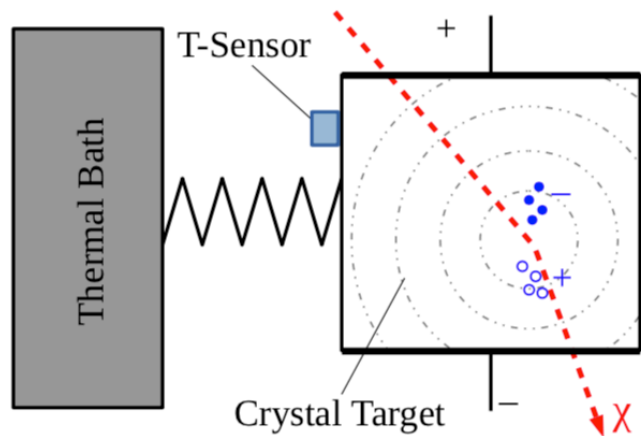
A nice introduction in Particle Astrophysics by D. H. Perkins (oxford master series in physics)

Amazing Drawing thanks to Dr. Laura Manenti! lm189@nyu.edu

SIGNATURES OF A DARK MATTER RECOIL

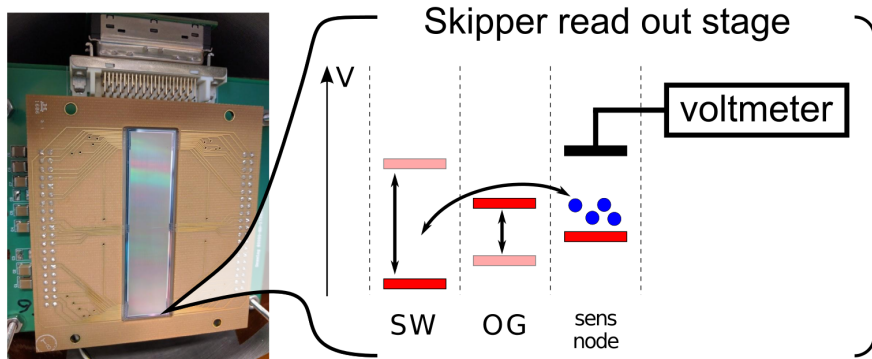


DETECTOR TECHNOLOGY EXAMPLES

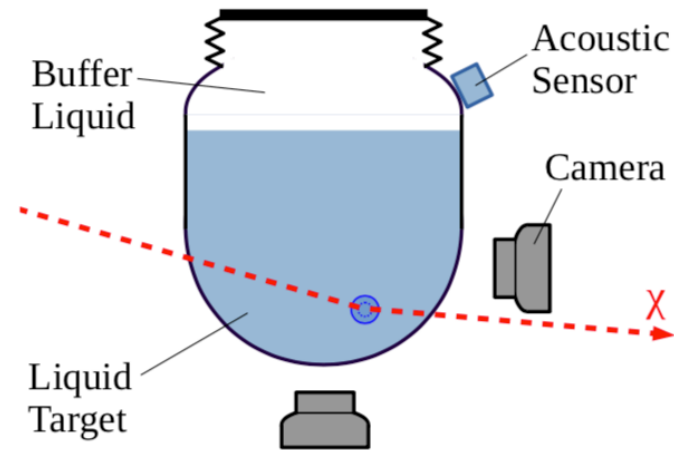


LIQUID NOBLE GAS 1- AND 2-PHASE TPCS

CRYOGENIC DETECTOR: HEAT & IONISATION



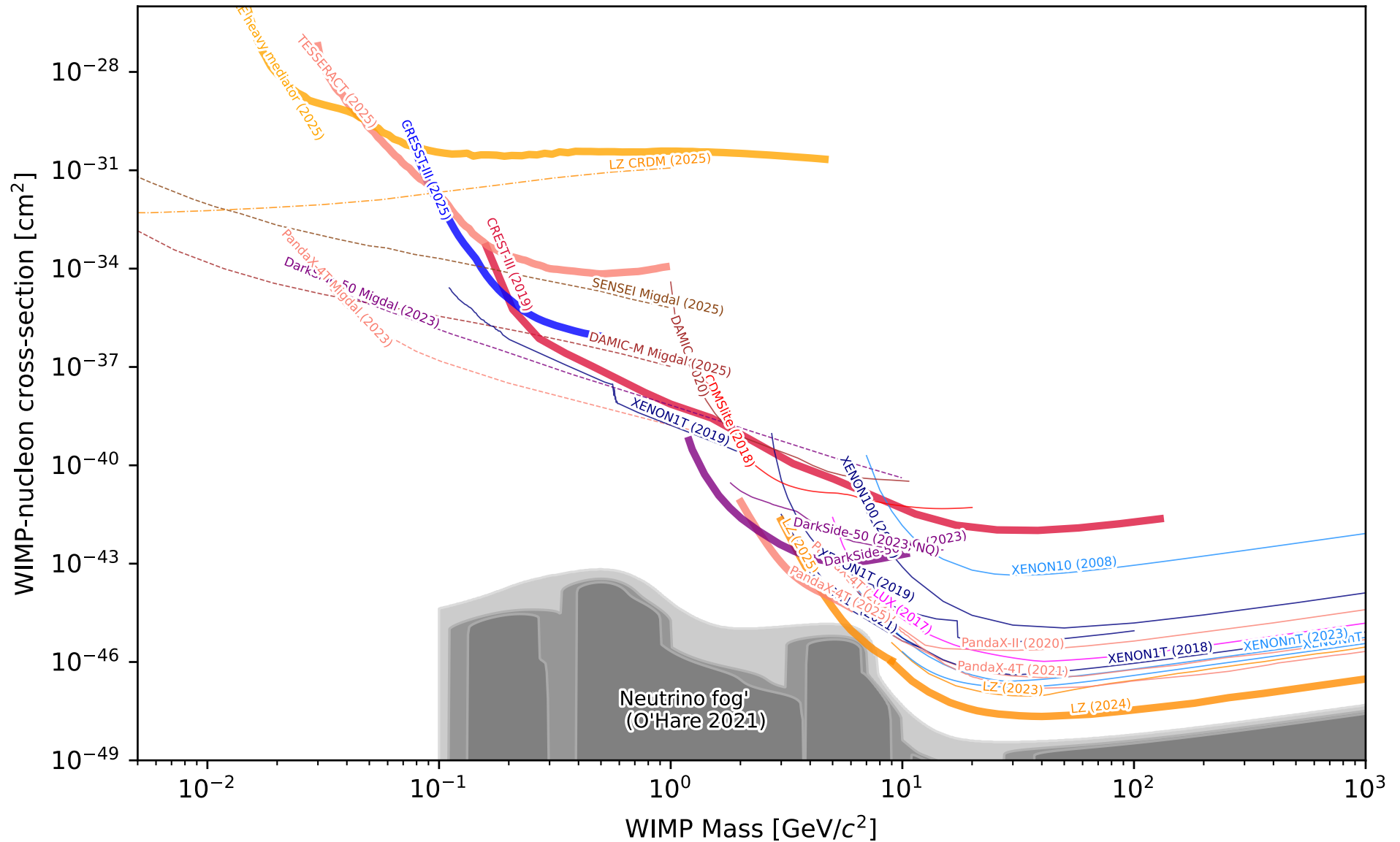
CCD READ OUT REPEATEDLY PER-PIXEL



BUBBLE CHAMBER

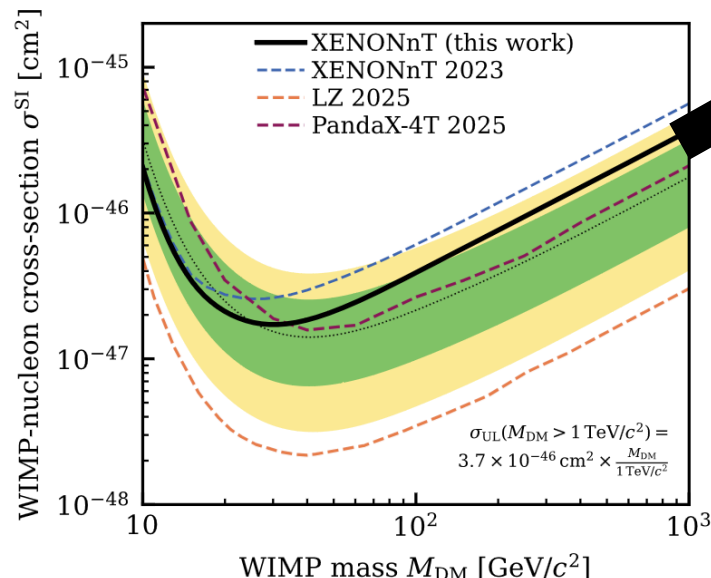
Figures except SENSEI from: Marc Schumann, "Direct Detection of WIMP Dark Matter: Concepts and Status". In: J. Phys. G 46:10 (2019), p. 103003. doi: 10.1088/1361-6471/ab2ea5. SENSEI illustration from <https://sensei-skipper.github.io/#SkipperCCD>

THE WIMP PARAMETER SPACE



WHY TRUNCATE AT 1 TeV/c²?

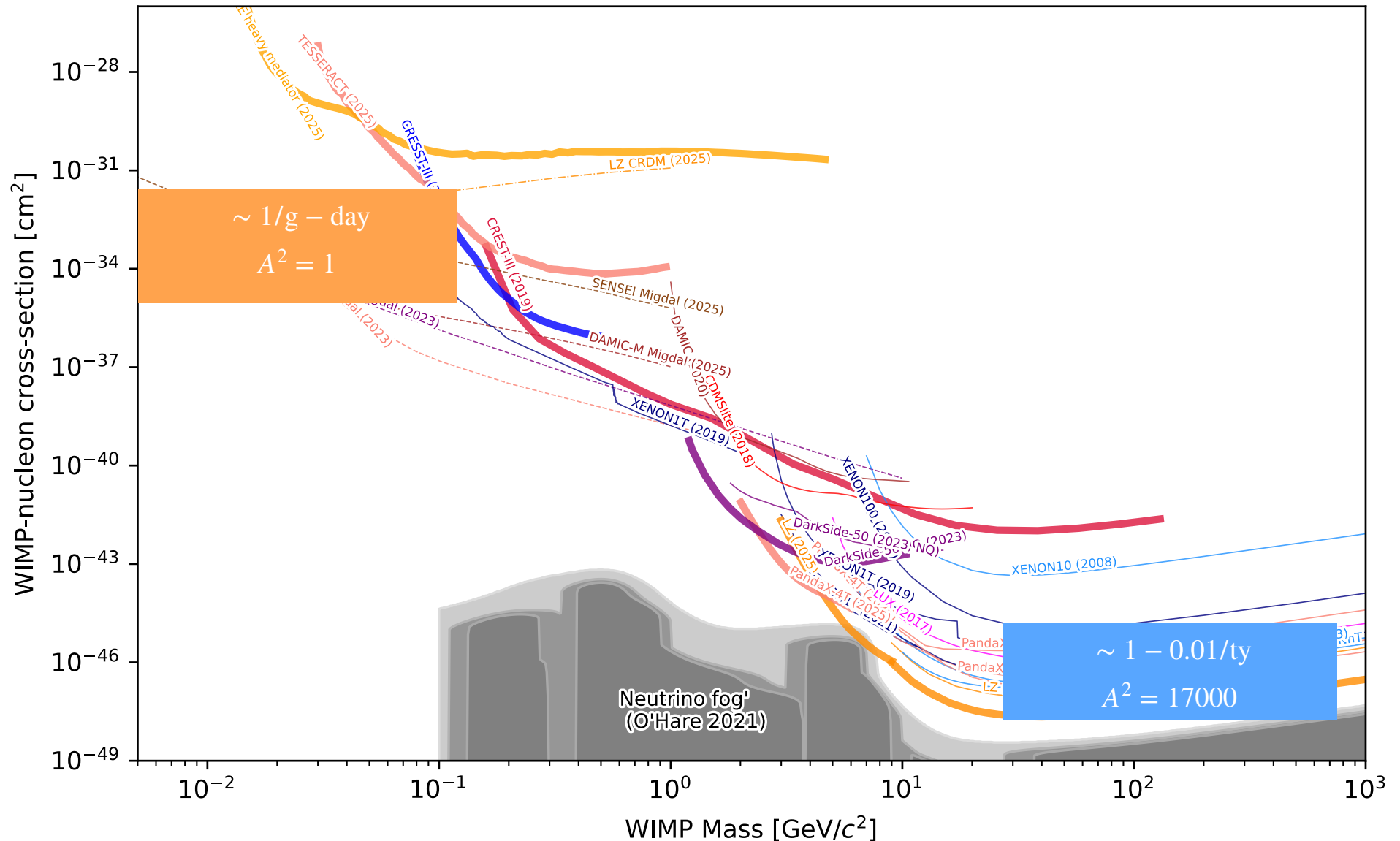
- Recall the maximum energy and form-factor calculations—once $m_T \lll m_\chi$, the energy of the collision is independent of m_χ
- The only dependence is therefore that as m_χ increases, the number density and therefore the flux decreases as $1/m_\chi$
- At masses greater than $\sim 340 \text{ TeV}/c^2$, the s-wave contribution is non-unitary
- At masses above $\sim 100 \text{ TeV}/c^2$, dark matter may start to form bound states that affect the relic abundance



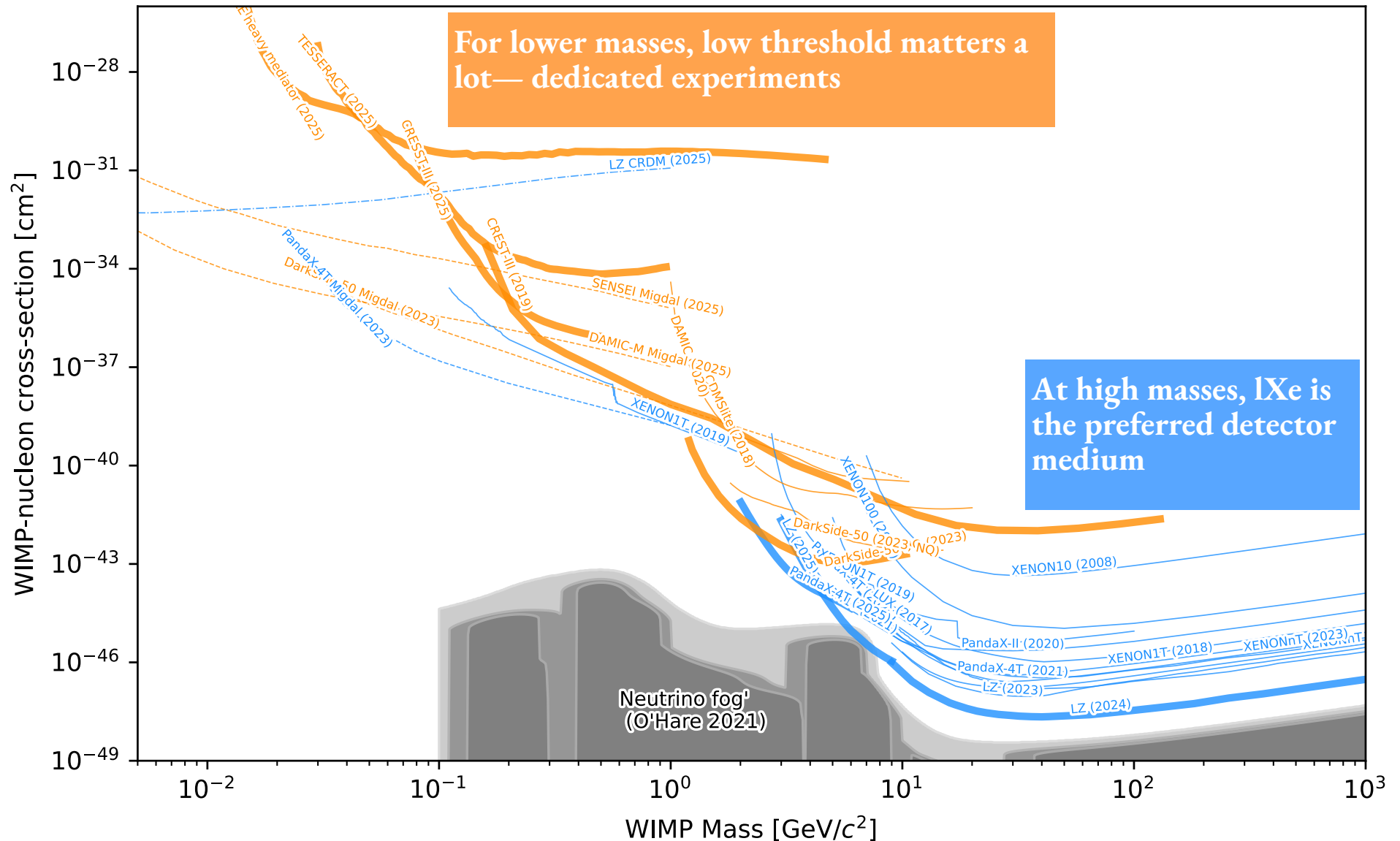
WIMP Dark Matter Search using a 3.1 Tonne-Year Exposure of the XENONnT Experiment
<https://arxiv.org/pdf/2502.18005>
 TeV-Scale Thermal WIMPs: Unitarity and its Consequences
<https://arxiv.org/pdf/1904.11503>

$$\sigma_{\text{UL}}(M_{\text{DM}} > 1 \text{ TeV}/c^2) = 3.7 \times 10^{-46} \text{ cm}^2 \times \frac{M_{\text{DM}}}{1 \text{ TeV}/c^2}$$

THE WIMP PARAMETER SPACE

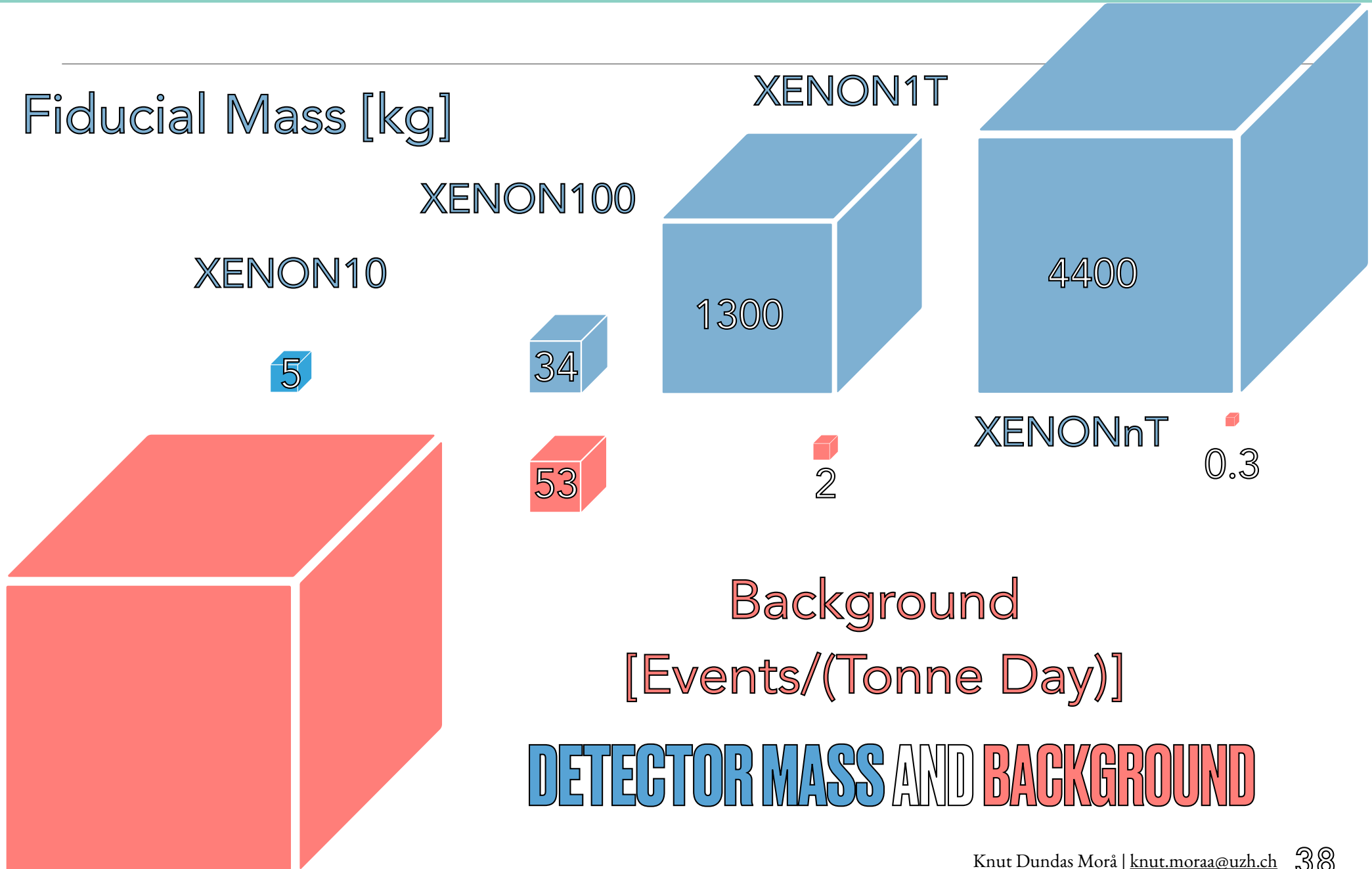


THE WIMP PARAMETER SPACE



LIQUID XENON HAS SCALED UP RAPIDLY

Fiducial Mass [kg]



Background
[Events/(Tonne Day)]

DETECTOR MASS AND BACKGROUND

XENON AS A DETECTOR MEDIUM

- **High atomic mass** ~ 130 enhances elastic scattering
- **Dense**; 2.85g/cm^3 , which allows a compact detector for a large mass
- **Liquid between -170 and -180 K**
- **No problematic radioactive isotopes**
- **Chemically inert**, allowing purification
- **Scintillation in vacuum-UV**, 174nm , which is well-matched to photosensors

Isotopes of xenon ($_{54}\text{Xe}$)

	Main isotopes ^[1]		Decay	
	abundance	half-life ($t_{1/2}$)	mode	product
¹²⁴Xe	0.095%	1.1×10^{22} y ^[2]	$\epsilon\epsilon$	¹²⁴ Te
¹²⁵Xe	synth	16.87 h	β^+	¹²⁵ I
¹²⁶Xe	0.089%	stable		
¹²⁷Xe	synth	36.342 d	ϵ	¹²⁷ I
¹²⁸Xe	1.91%	stable		
¹²⁹Xe	26.4%	stable		
¹³⁰Xe	4.07%	stable		
¹³¹Xe	21.2%	stable		
¹³²Xe	26.9%	stable		
¹³³Xe	synth	5.2474 d	β^-	¹³³ Cs
¹³⁴Xe	10.4%	stable		
¹³⁵Xe	synth	9.14 h	β^-	¹³⁵ Cs
¹³⁶Xe	8.86%	2.18×10^{21} y	$\beta^-\beta^-$	¹³⁶ Ba

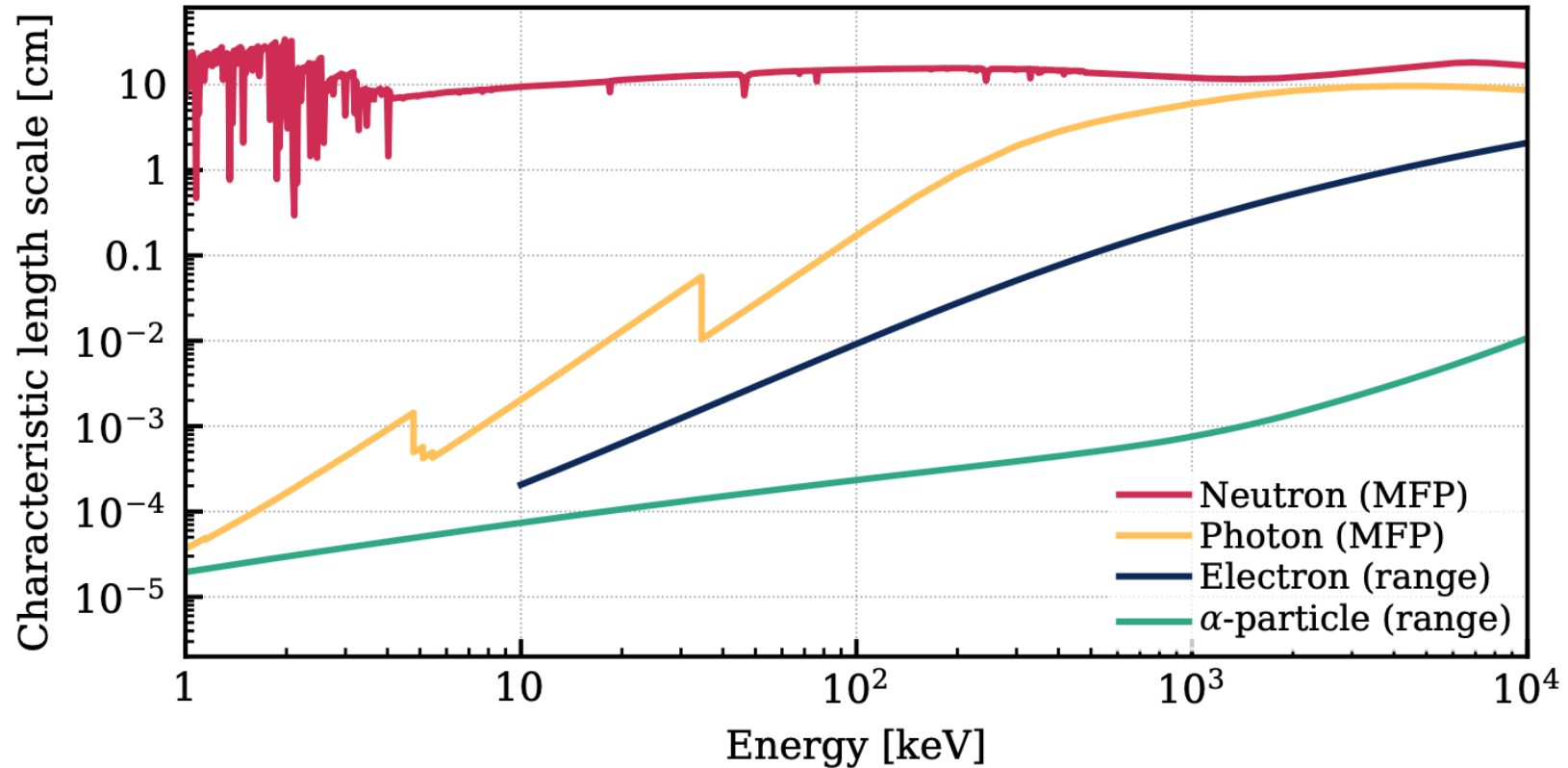
Standard atomic weight $A_r^\circ(\text{Xe})$

131.293 ± 0.006 ^[3]
 131.29 ± 0.01 (abridged)^[4]

[view](#) · [talk](#) · [edit](#)

https://en.wikipedia.org/wiki/Isotopes_of_xenon

XENON SELF-SHIELDING



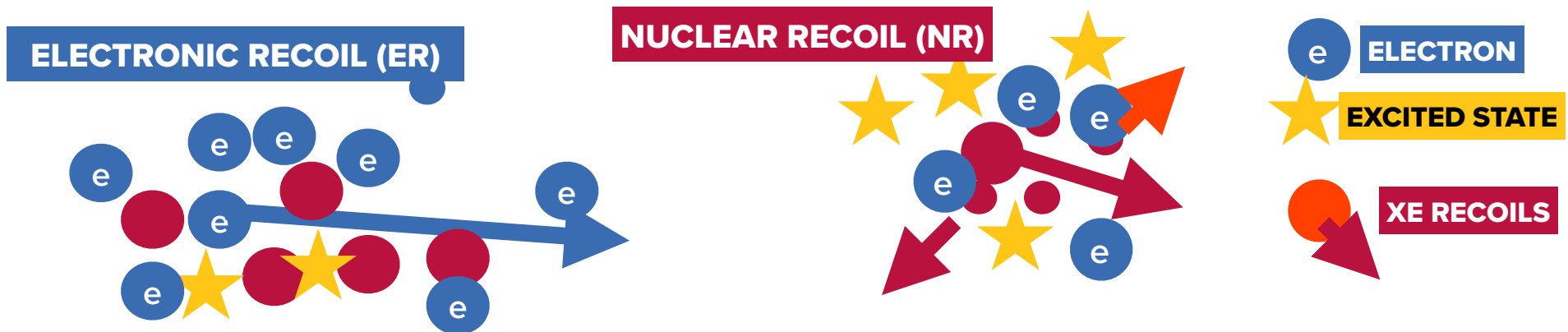
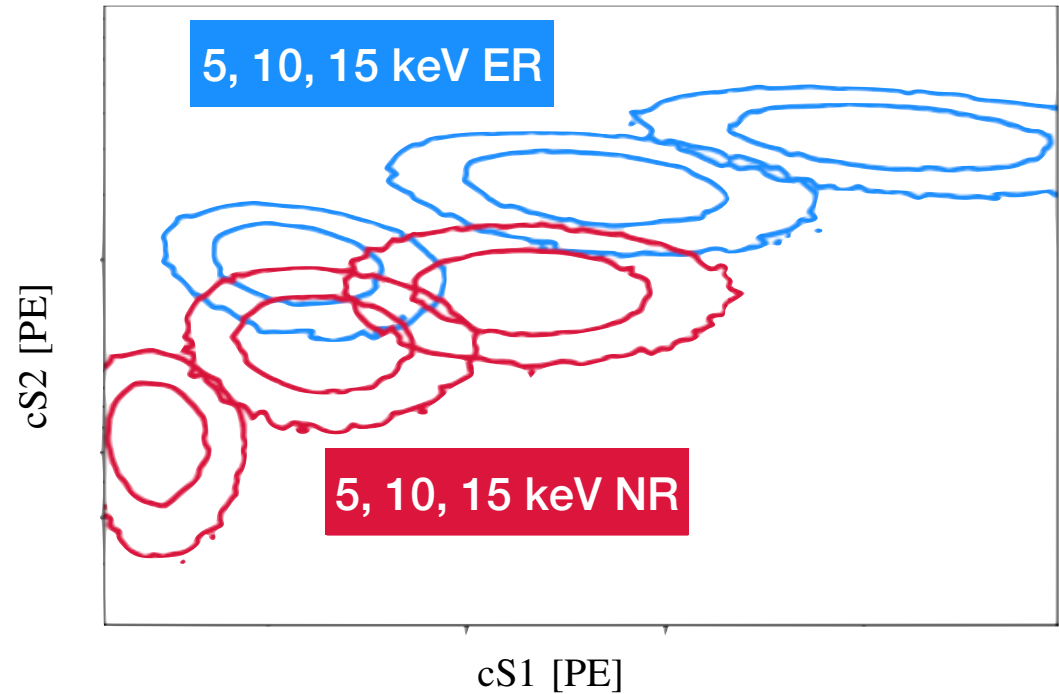
- In the dark matter region of interest, only neutrons have a characteristic length scale longer than a few mm

XENON AS A DETECTOR COST DRIVER

- **The cost of Xe is**
 $\sim (0.5 - 5) \times 10^6$ USD/tonne
 - **Fluctuating cost makes planning difficult**
 - **On the other hand, the Xe is available after the detector is done— rent, not buy**
- **10^{-7} of the air— you need two cubic tonnes of air to make a tonne.**
- **Extracted as a by-product of oxygen/nitrogen liquification**

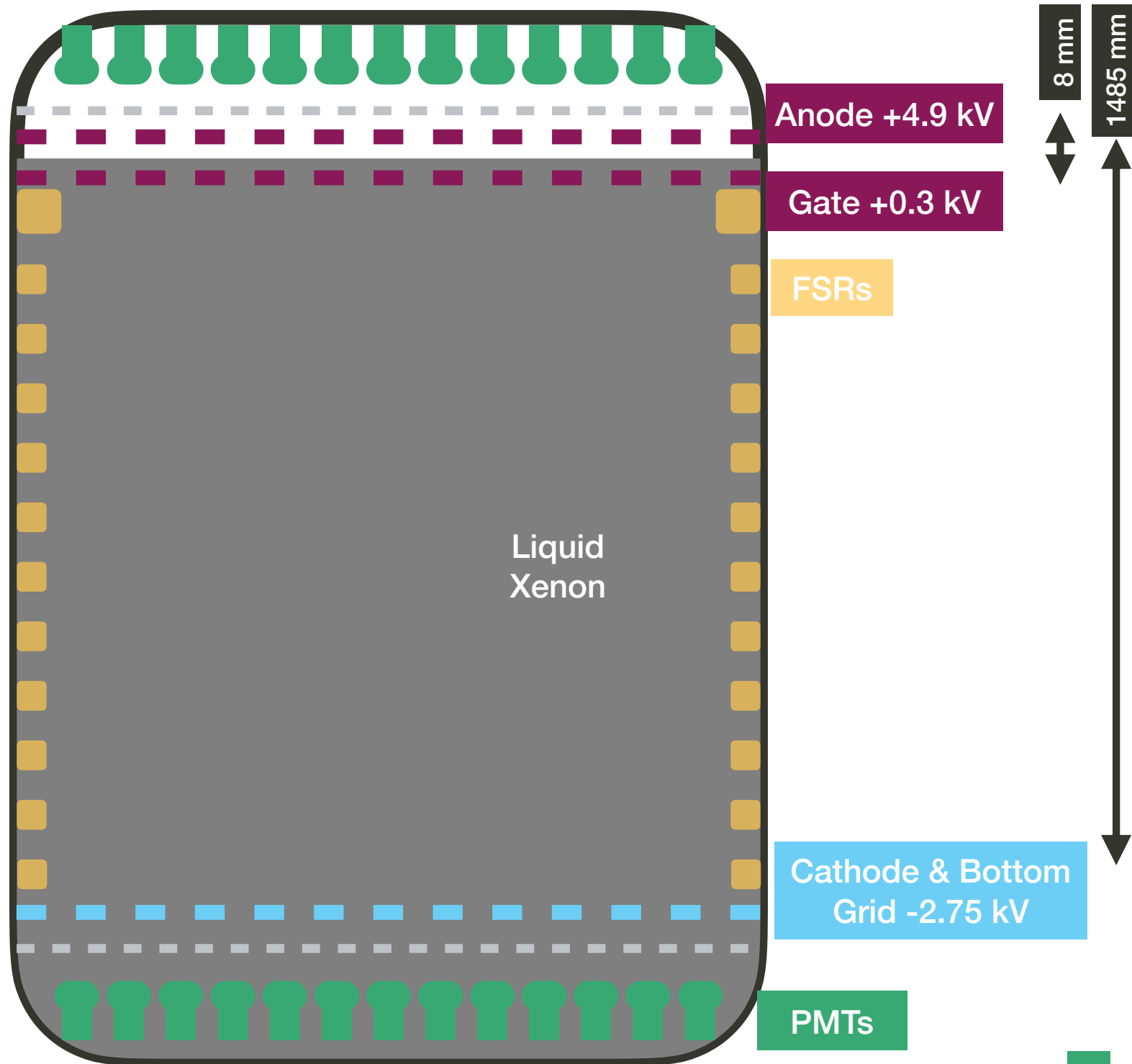
OBSERVABLE SIGNALS IN LIQUID XENON

- Recoiling xenon atoms or electrons in the liquid cause ionization and excites xenon atoms, which will yield scintillation light
- Atomic motion, heat, is lost, so that less energy is observed when the nucleus recoils
- The ionization and scintillation signals are observed in two-phase LXe TPCs



TWO-PHASE TPC

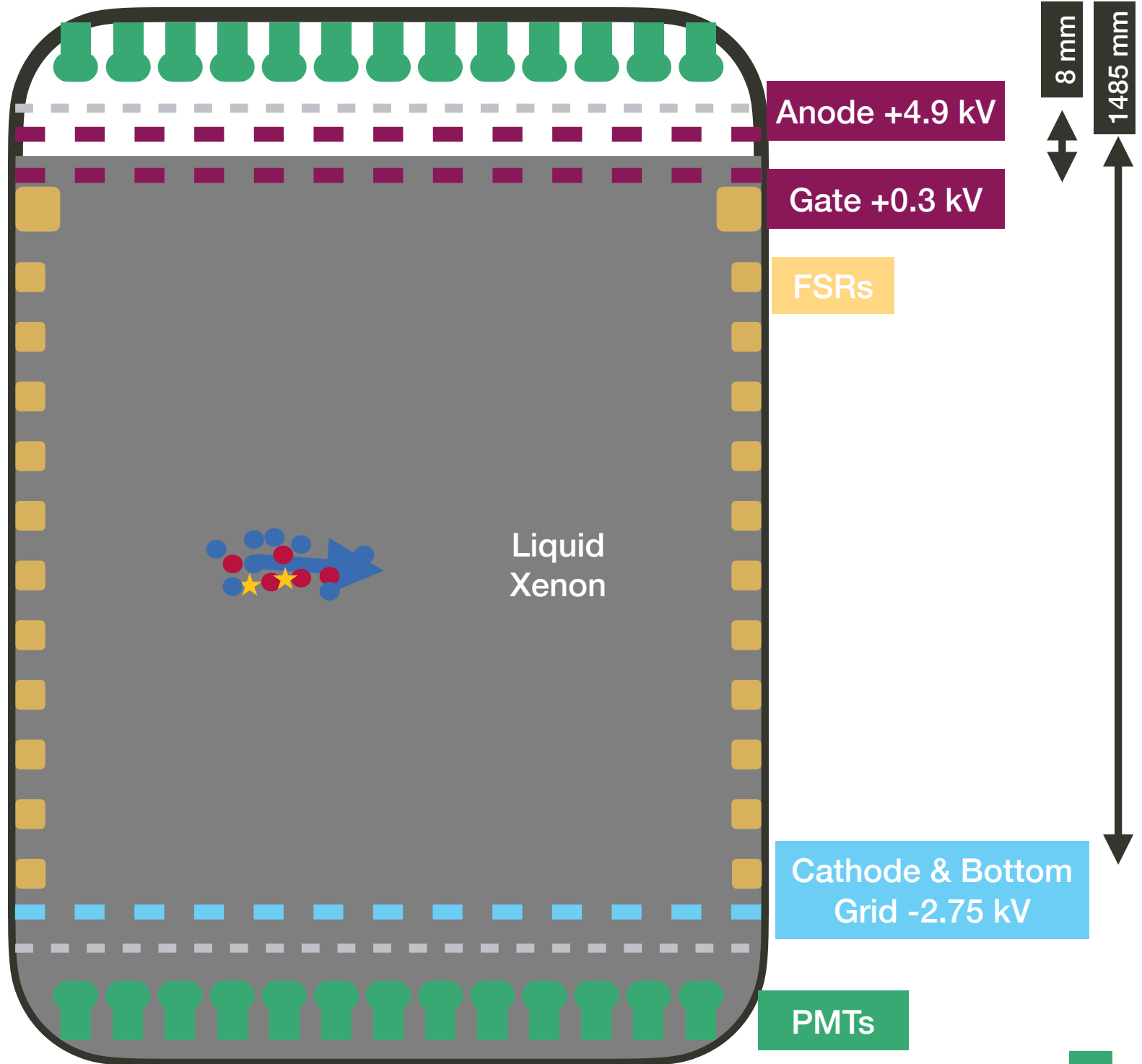
With XENONnT SR0 numbers



TWO-PHASE TPC

With XENONnT SR0 numbers

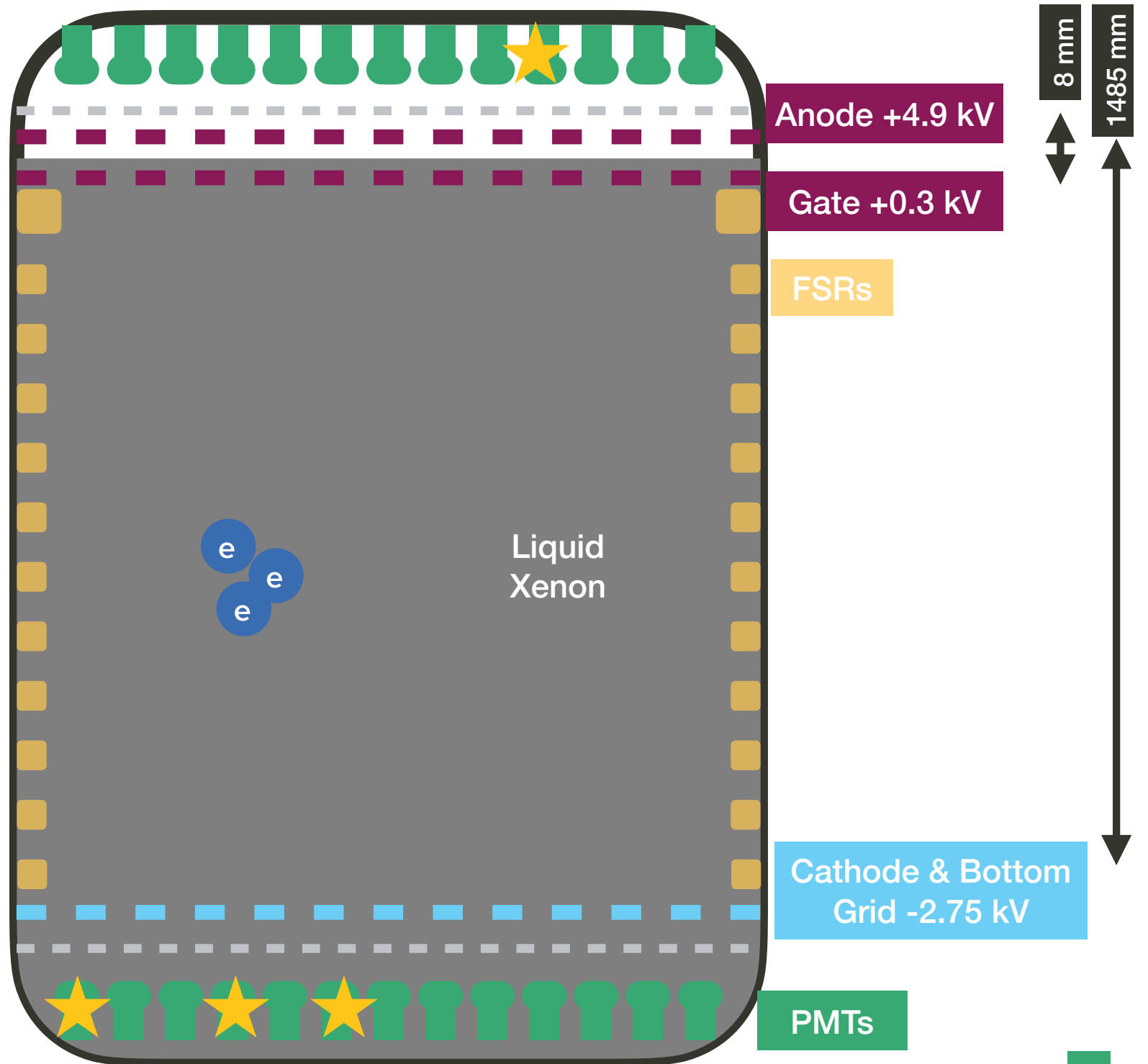
An Interaction deposits energy, scintillation light and charge is liberated



TWO-PHASE TPC

With XENONnT SR0 numbers

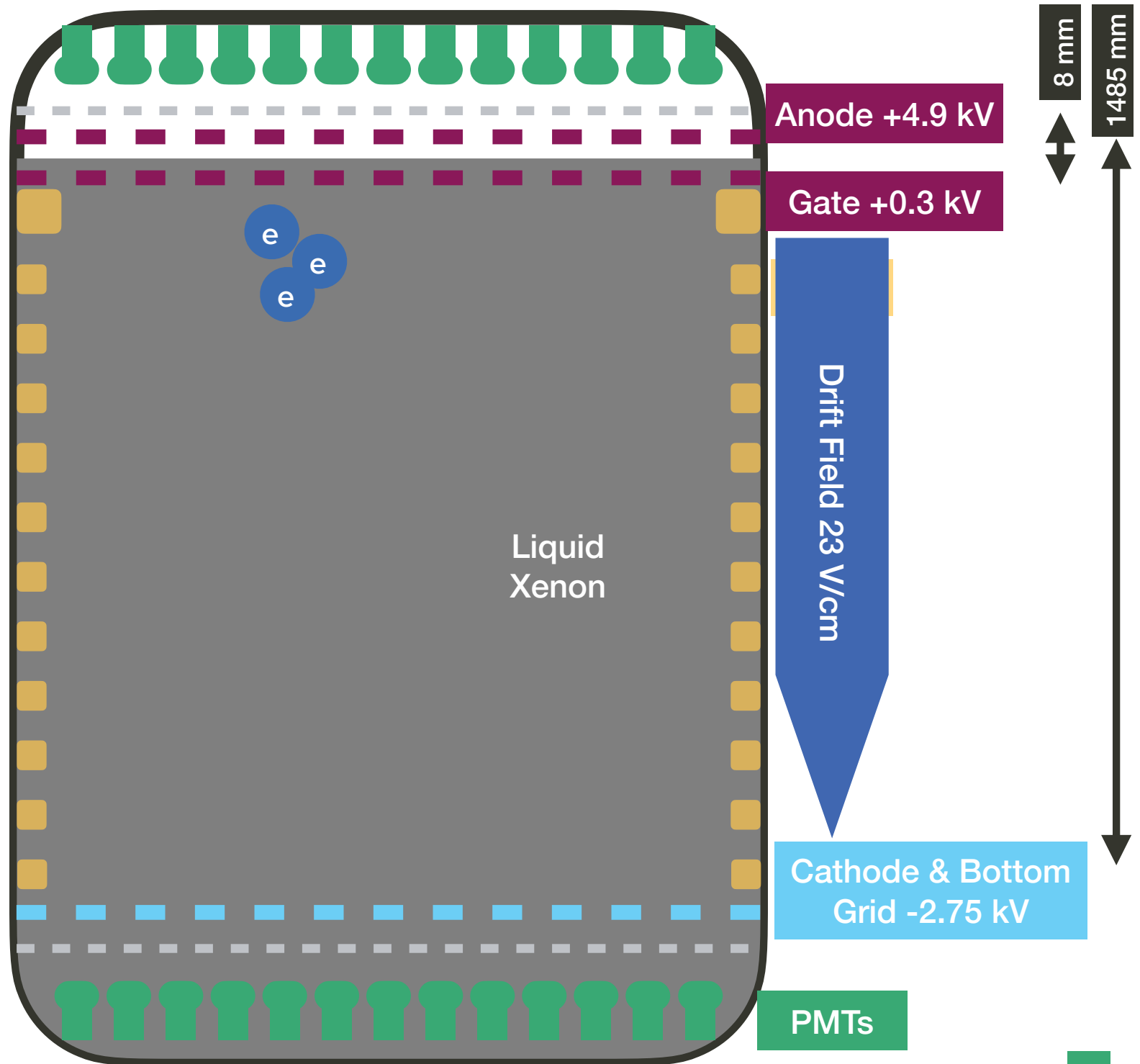
- An Interaction deposits energy, scintillation light and charge is liberated
- S1 signal reaches photomultipliers



TWO-PHASE TPC

With XENONnT SR0 numbers

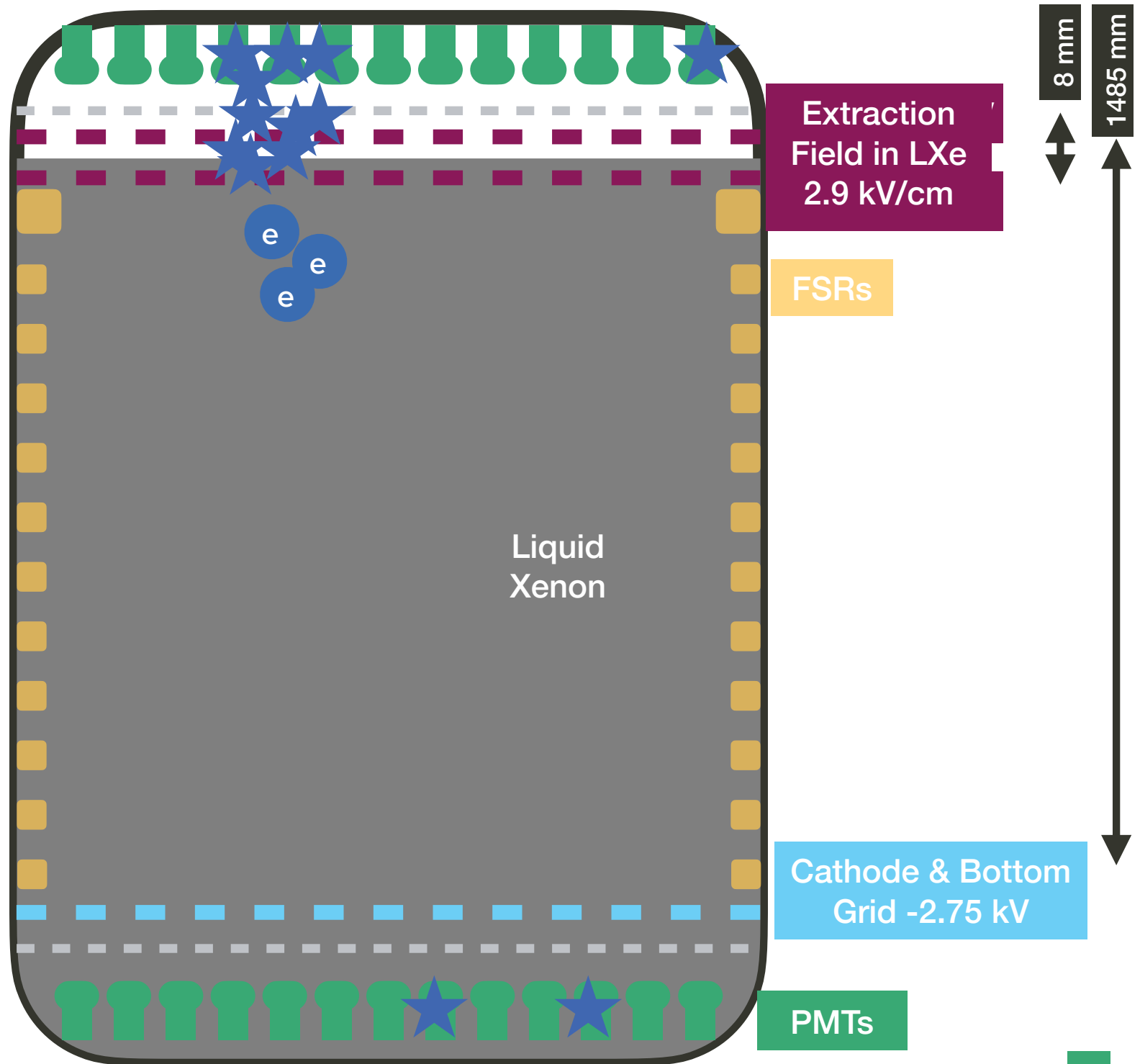
- An Interaction deposits energy, scintillation light and charge is liberated
- S1 signal reaches photomultipliers
- Electrons drift to the surface



TWO-PHASE TPC

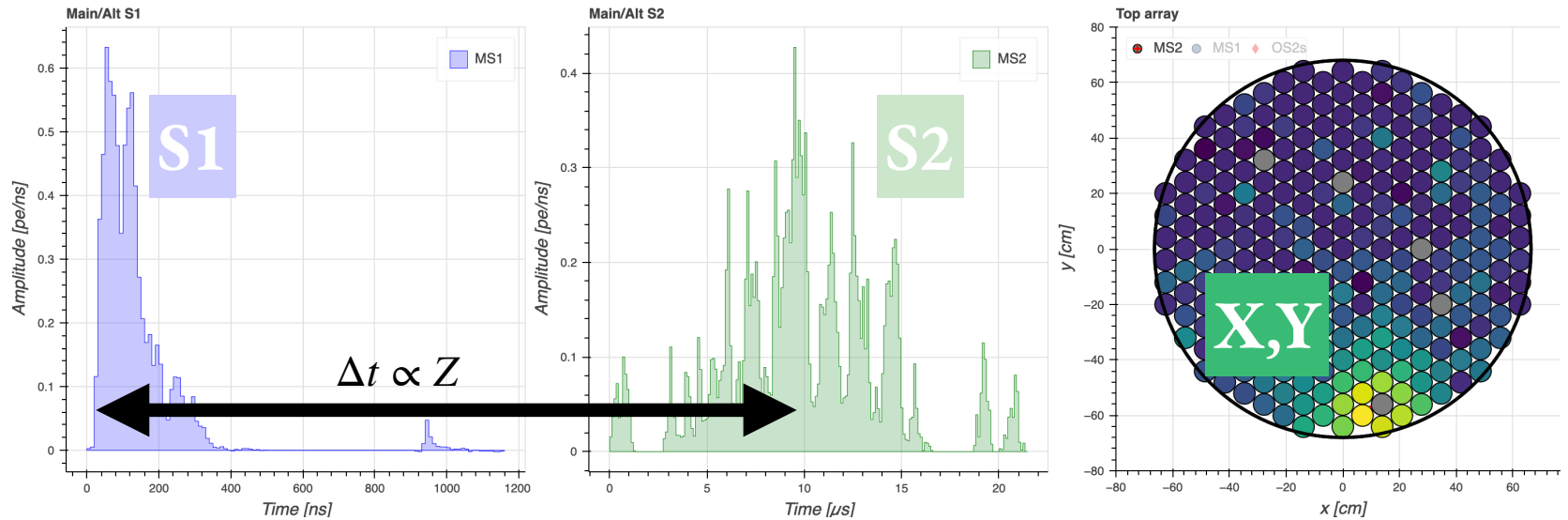
With XENONnT SR0 numbers

- An Interaction deposits energy, scintillation light and charge is liberated
- S1 signal reaches photomultipliers
- Electrons drift to the surface
- The extraction field pulls the electrons to the gas phase where they make more scintillation light

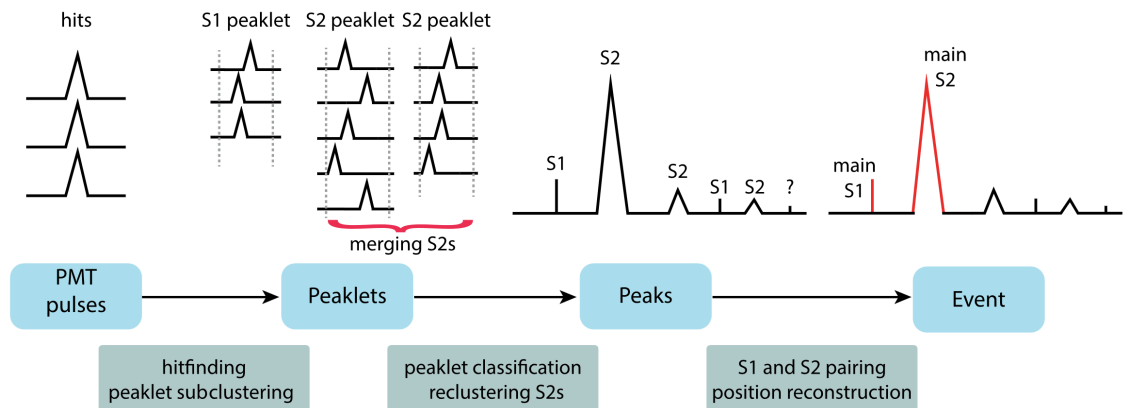


OBSERVABLES

Event 17639 from run 020301
 Recorded at 2021-05-24 T16:25:20 UTC, 685013576 ns - 687737380 ns



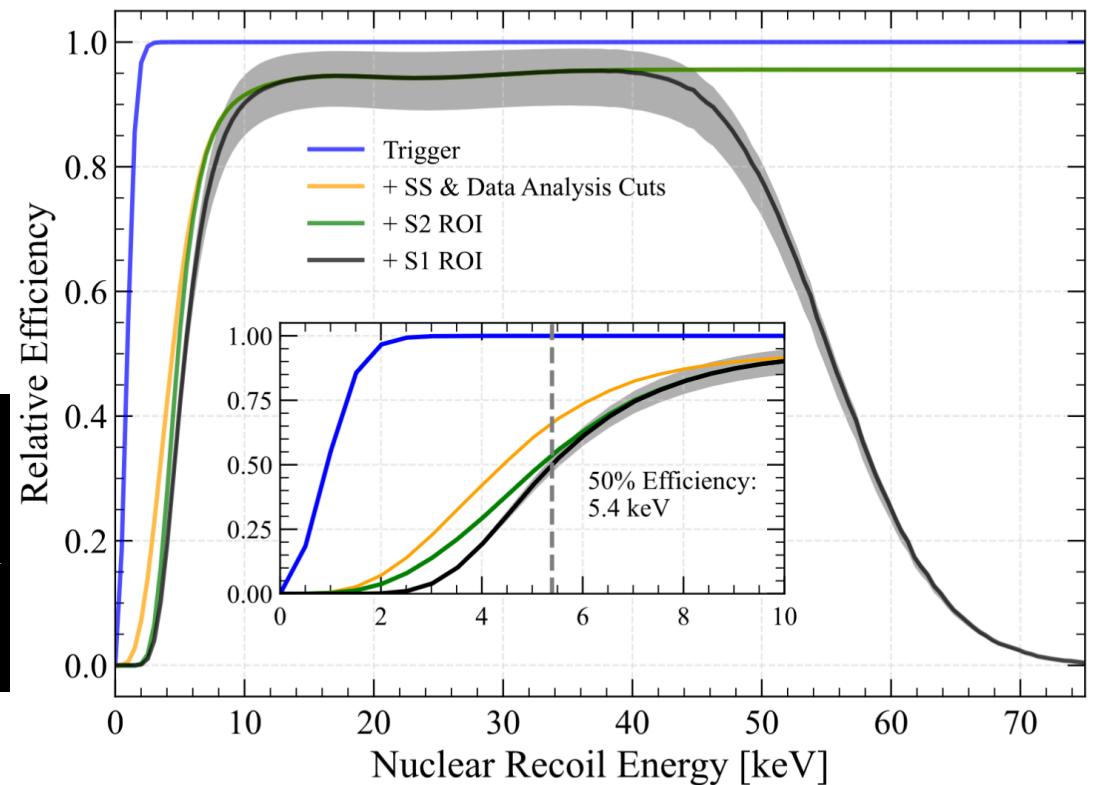
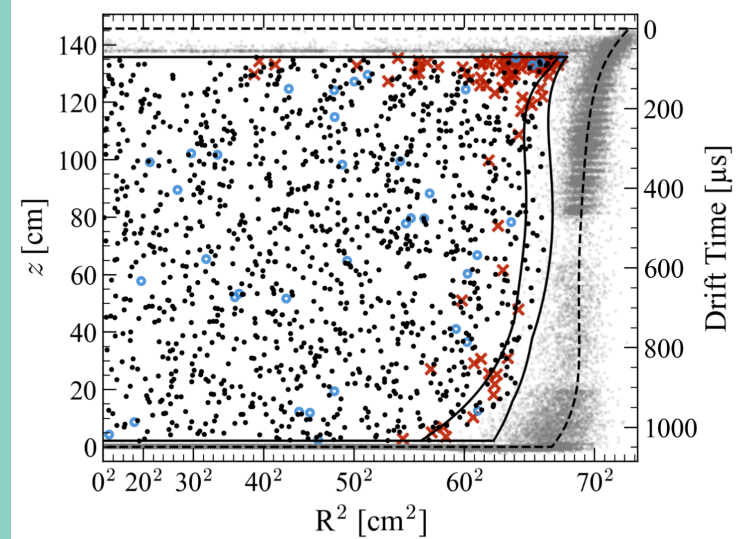
- “Waveforms”— the time trace of the signal in the photosensors is used to reconstruct S1, S2, the arrival time between them and the XY position



SELECTIONS

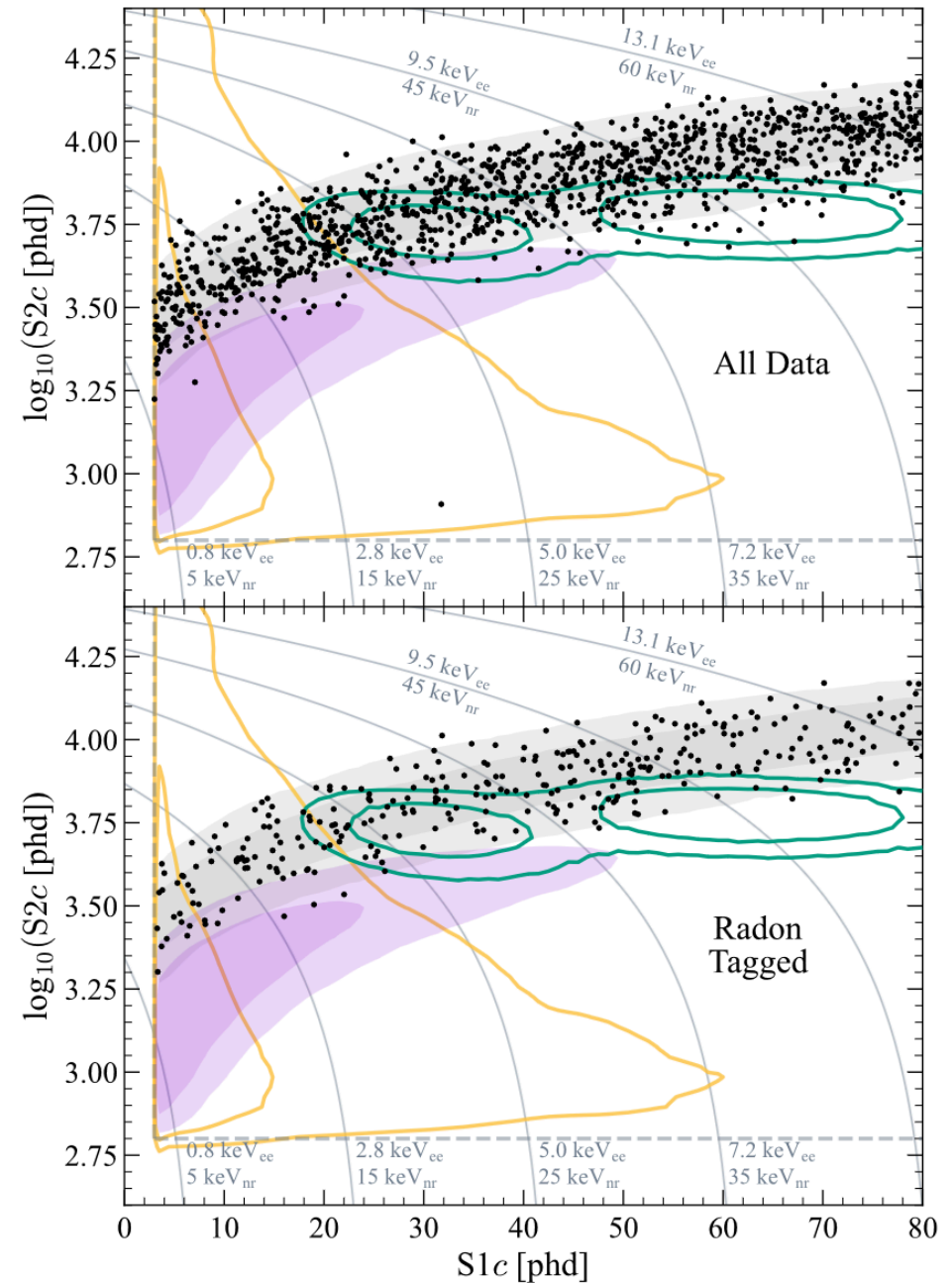
- The 3D reconstruction allows the detector to select away very background-dominated regions
- Together with the requirement to see a signal at all (“trigger”), this gives the efficiency of the detector — the probability to reconstruct an event

$$\langle N_{\text{signal}} \rangle = \Delta t \times M_{\text{detector}} \int \epsilon(E_{\text{recoil}}) \times \frac{dR}{dE_{\text{recoil}}} dE_{\text{recoil}}$$



SIGNAL AND BACKGROUND MODELS

- The selected data is analysed in $S1, S2$ space, and is dominated by electronic recoil events (gray band)
- A number of background shapes are also included in the full background model
- Together with a WIMP signal model, falling in the nuclear recoil region, this makes up the statistical model of the data
 - This model depends on the WIMP mass and interaction
- With the statistical model, we can construct best-fits, significance and confidence interval results for each WIMP model.

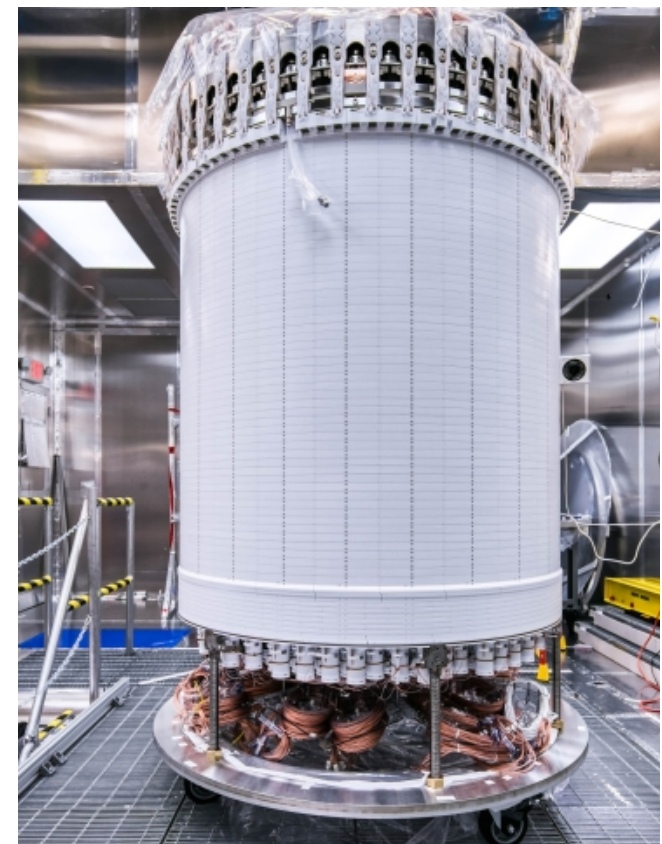
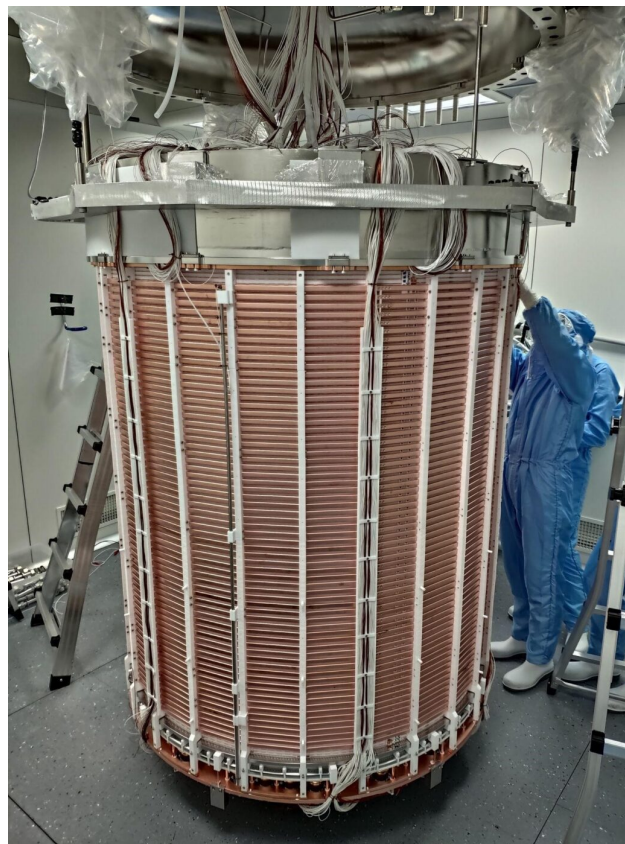


THREE DETECTORS ALIKE IN DIGNITY

PandaX-4T
3.7 ton LXe
2020-2026

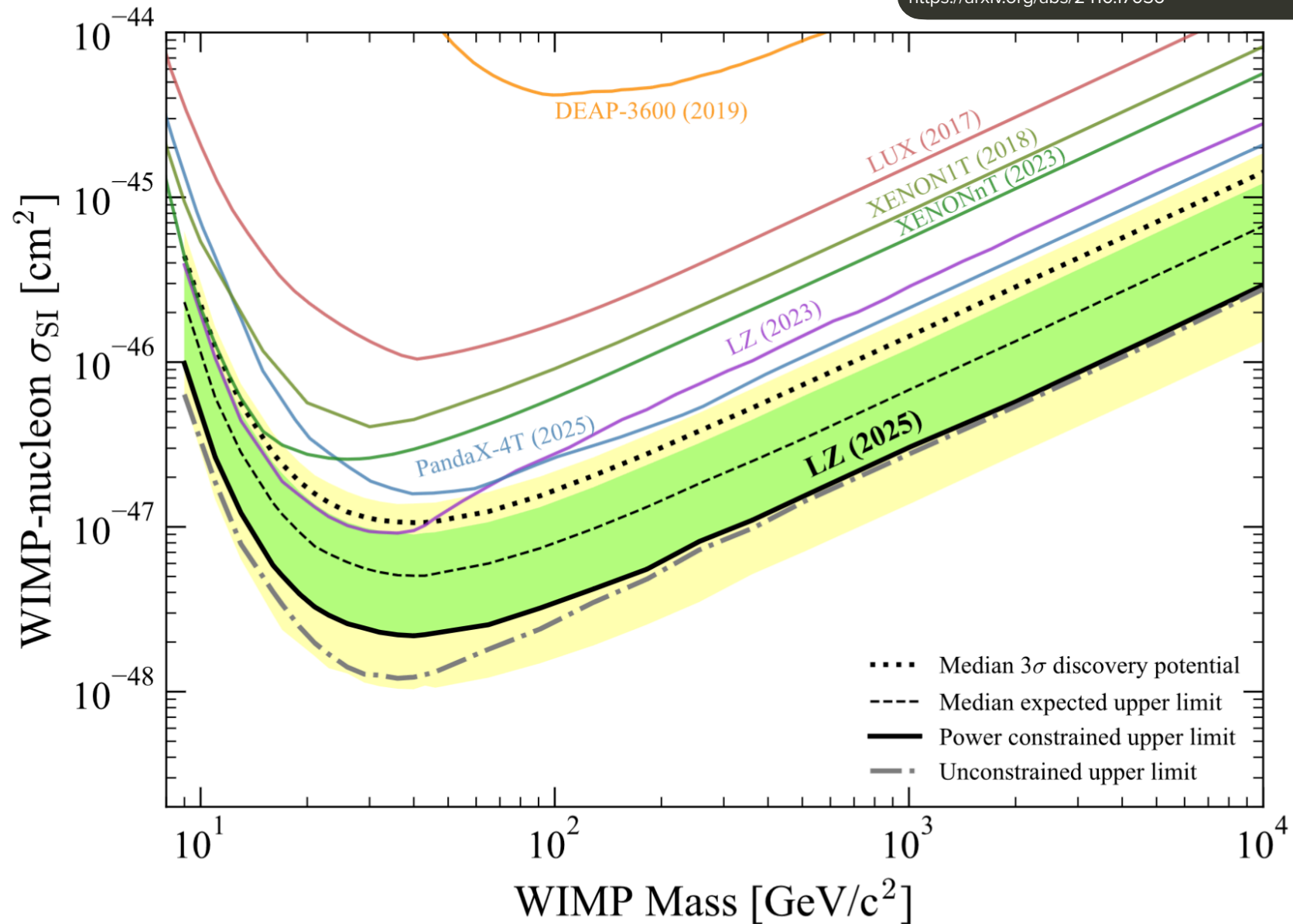
XENONnT
5.9 ton LXe
2021-ongoing

LZ (Lux-Zepplin)
8 ton LXe
2021-ongoing

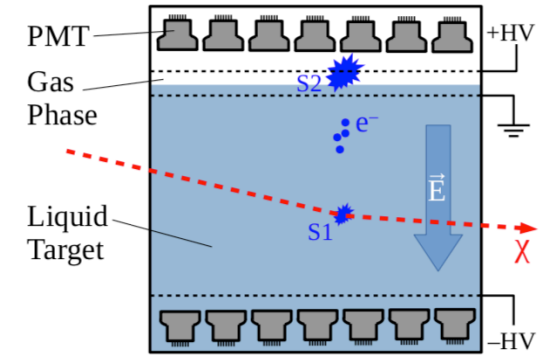
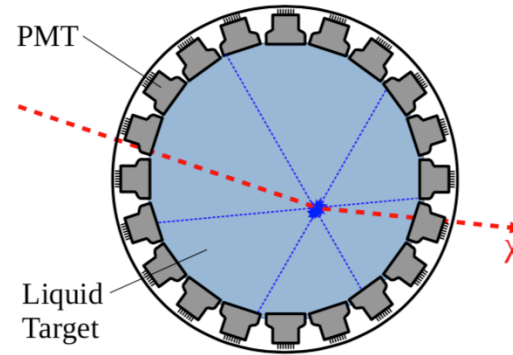
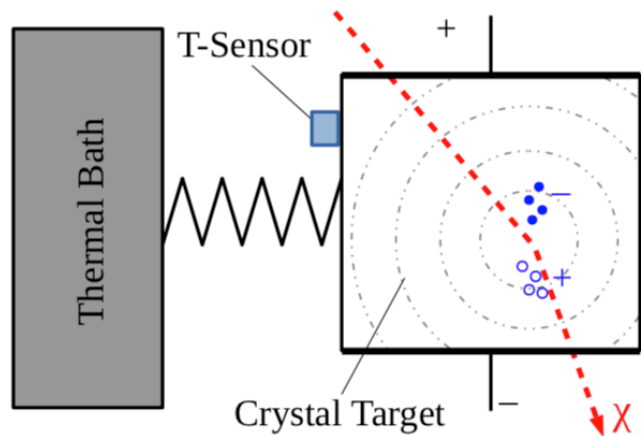


SO FAR, VERY GOOD NULL RESULTS

Dark Matter Search Results from 4.2 Tonne-Years of Exposure of the LUX-ZEPLIN (LZ) Experiment
<https://arxiv.org/abs/2410.17036>

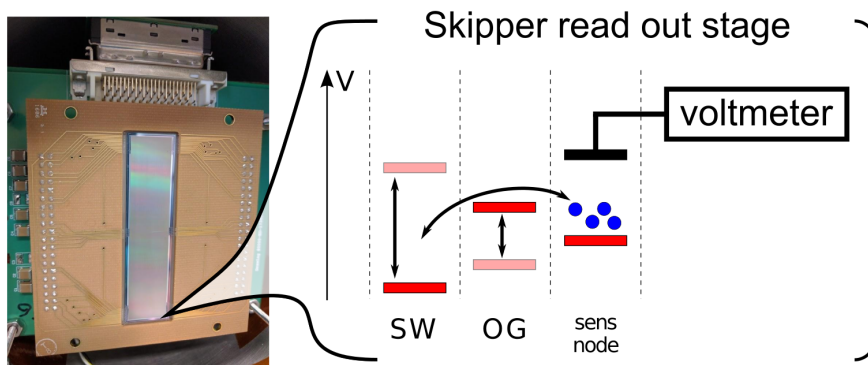


MANY DETECTOR TECHNOLOGIES ARE MOTIVATED BY A DESIRE TO COMPLEMENT XENON DETECTORS

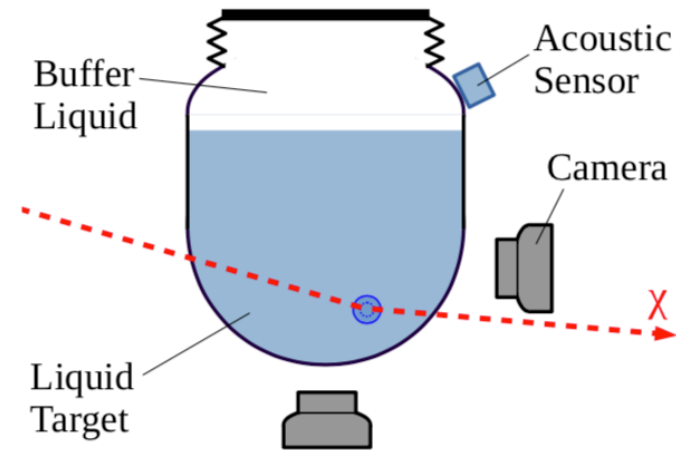


LIQUID NOBLE GAS 1- AND 2-PHASE TPCS

CRYOGENIC DETECTOR: HEAT & IONISATION



CCD READ OUT REPEATEDLY PER-PIXEL

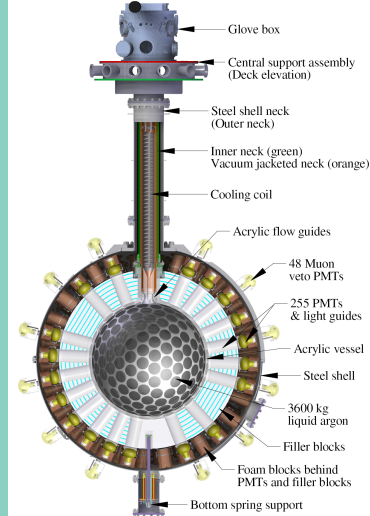
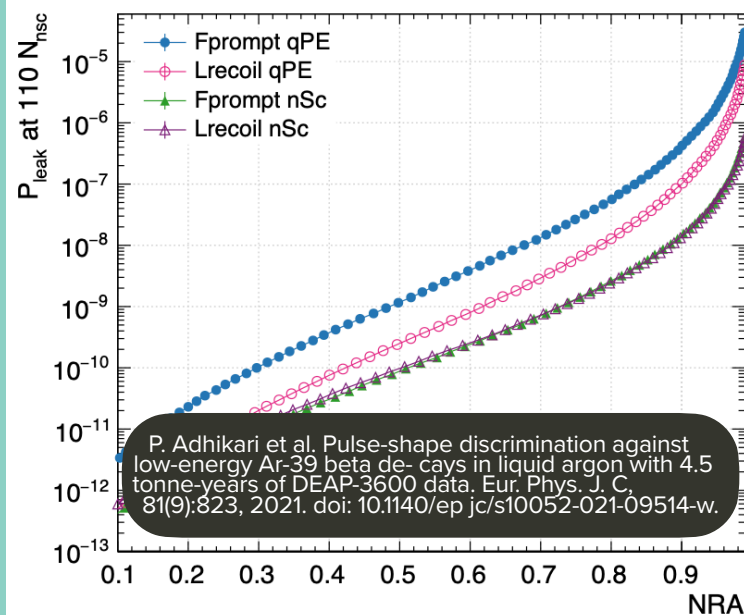


BUBBLE CHAMBER

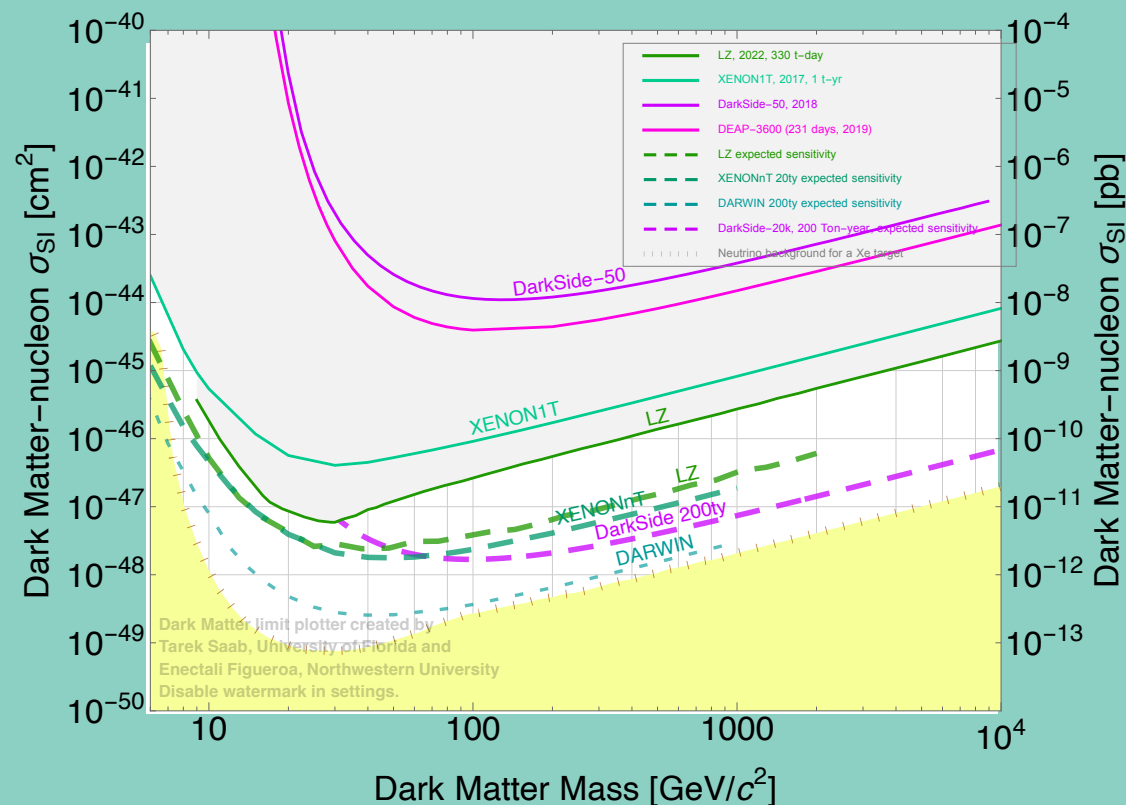
Figures except SENSEI from: Marc Schumann. "Direct Detection of WIMP Dark Matter: Concepts and Status". In: J. Phys. G 46.10 (2019), p. 103003. doi: 10.1088/1361-6471/ab2ea5. SENSEI illustration from <https://sensei-skipper.github.io/#SkipperCCD>

ARGON DETECTORS

- Argon shares many of the positive properties of xenon, but is lighter
- Lower scintillation wavelength, $\lambda = 128\text{nm}$
- Scintillation time structure allows the detector to resolve electronic recoils and electronic recoils from pulse shape alone
- Darkside-20K aims to achieve a nearly background-free ($\mu_{\text{bkg}} \ll 1$) 100 tonne-year exposure
- 20 tonne fiducial volume (surface background observed with Darkside-50)
- Underground argon will reduce the ^{39}Ar background, but projections still assume an ER-limited threshold of 30 keV



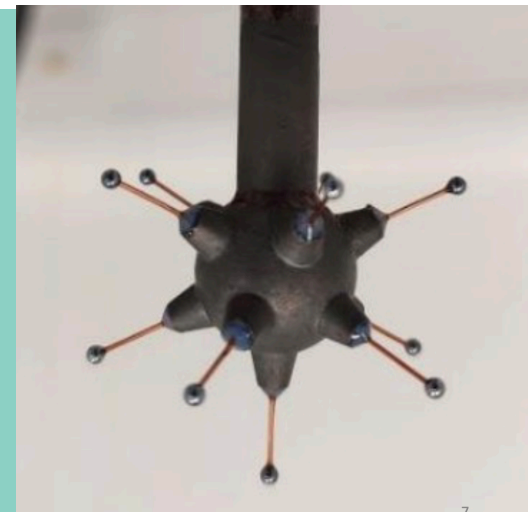
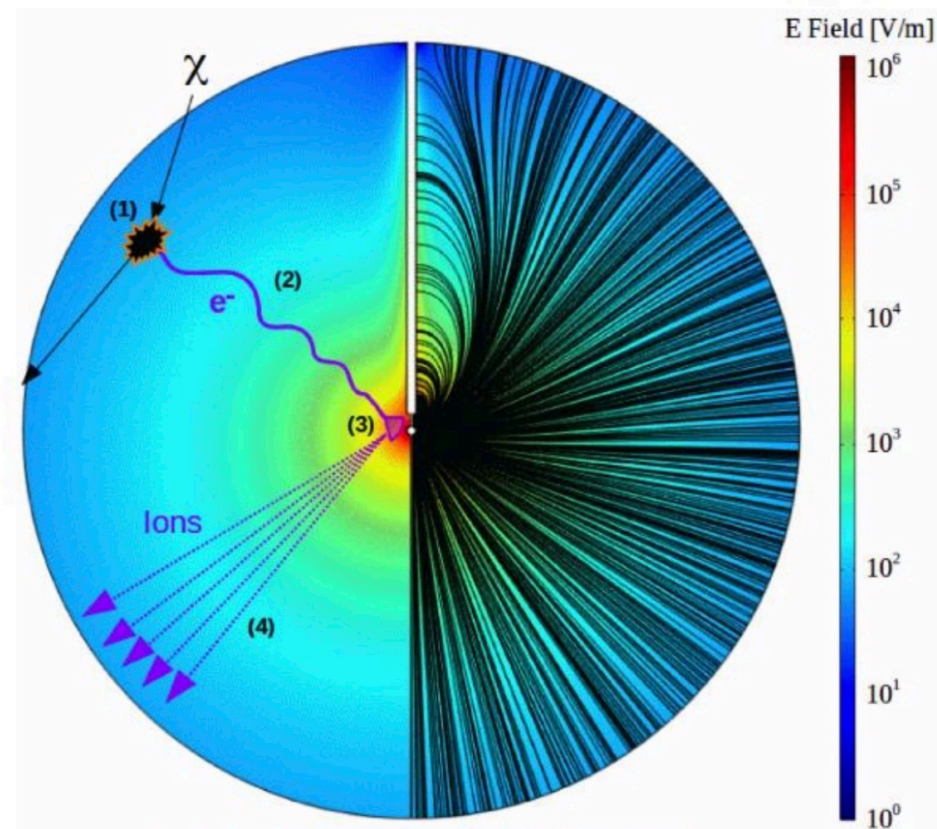
Schematic courtesy of DEAP-3600



C. E. Aalseth et al. DarkSide-20k: A 20 tonne two-phase LAr TPC for direct dark matter detection at LNGS. Eur. Phys. J. Plus, 133:131, 2018. doi: 10.1140/epjp/i2018-11973-4.

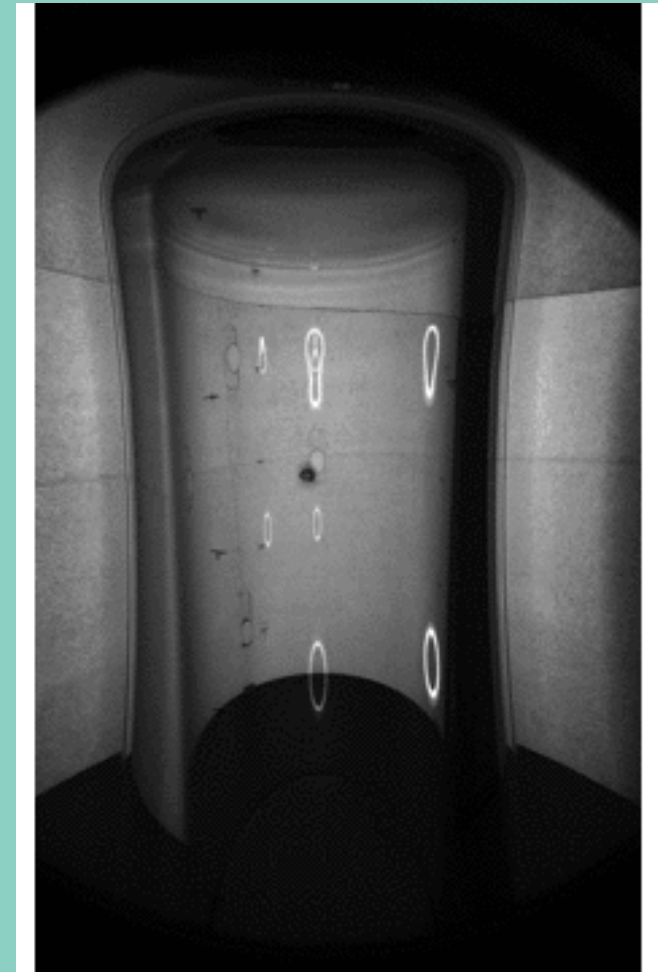
SINGLE-PHASE

- NEWS-G uses a spherical volume filled with a target gas (H, He, Ne) and a very strong electric field to read out events
- The very strong field near the central sensor/electrode causes a strong amplification of the signal
- Being able to search for electron-only events makes them, in principle, better optimised for ionization-only studies than the two-phase TPCs
- Good limits on spin-dependent interactions



BUBBLE CHAMBERS

- The PICO (PICASSO+COUPP) series of experiments use supercritical fluid bubble chambers to detect nuclear recoil heat depositions
- Electronic recoils produce less dense energy depositions, and do not form bubbles— intrinsic NR/ER discrimination
- Observables: images of bubbles, and the sound as the bubble bursts
 - neutrons can be distinguished by multiple tracks or spatially
 - Alpha bubbles are louder— 99+ % discrimination
- ~ 3keV energy thresholds in science mode, lower thresholds of $1.2 \pm 0.08\text{keV}$ can be achieved if you accept some ER leakage



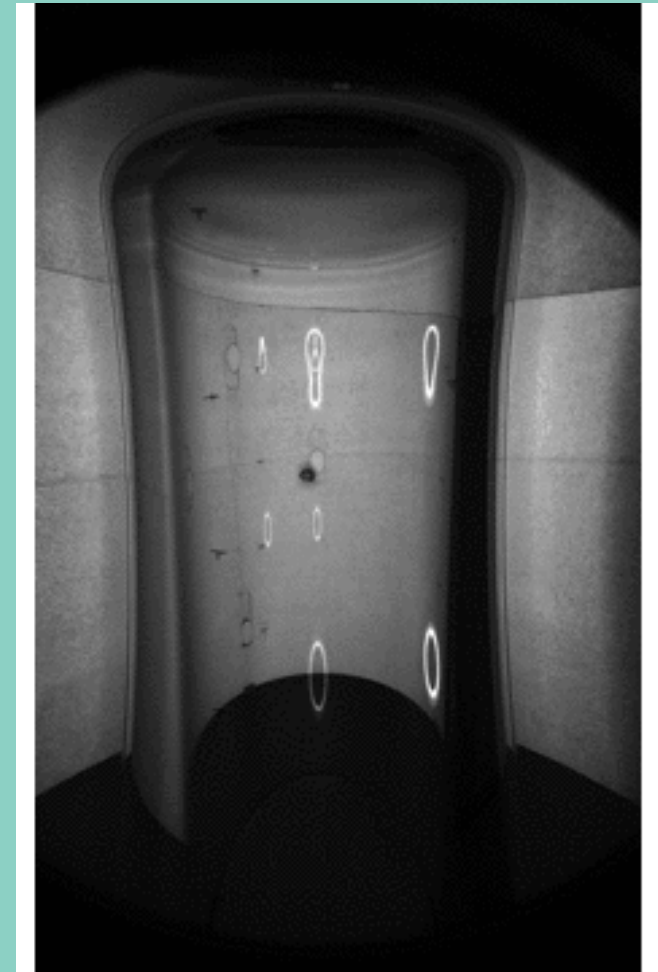
Single-bubble event captured during PICO-40L commissioning courtesy of the PICO experiment

F. Aubin et al. Discrimination of nuclear recoils from alpha particles with superheated liquids. *New J. Phys.*, 10:103017, 2008. doi: 10.1088/1367-2630/10/10/103017.

C. Amole et al. Dark Matter Search Results from the Complete Exposure of the PICO-60 C_3F_8 Bubble Chamber. *Phys. Rev. D*, 100(2):022001, 2019. doi: 10.1103/PhysRevD.100.022001.

BUBBLE CHAMBERS

- The PICO (PICASSO+COUPP) series of experiments use supercritical fluid bubble chambers to detect nuclear recoil heat depositions
- Electronic recoils produce less dense energy depositions, and do not form bubbles— intrinsic NR/ER discrimination
- Observables: images of bubbles, and the sound as the bubble bursts
 - neutrons can be distinguished by multiple tracks or spatially
 - Alpha bubbles are louder— 99+ % discrimination
- ~ 3keV energy thresholds in science mode, lower thresholds of $1.2 \pm 0.08\text{keV}$ can be achieved if you accept some ER leakage



Single-bubble event captured during PICO-40L commissioning courtesy of the PICO experiment

F. Aubin et al. Discrimination of nuclear recoils from alpha particles with superheated liquids. *New J. Phys.*, 10:103017, 2008. doi: 10.1088/1367-2630/10/10/103017.

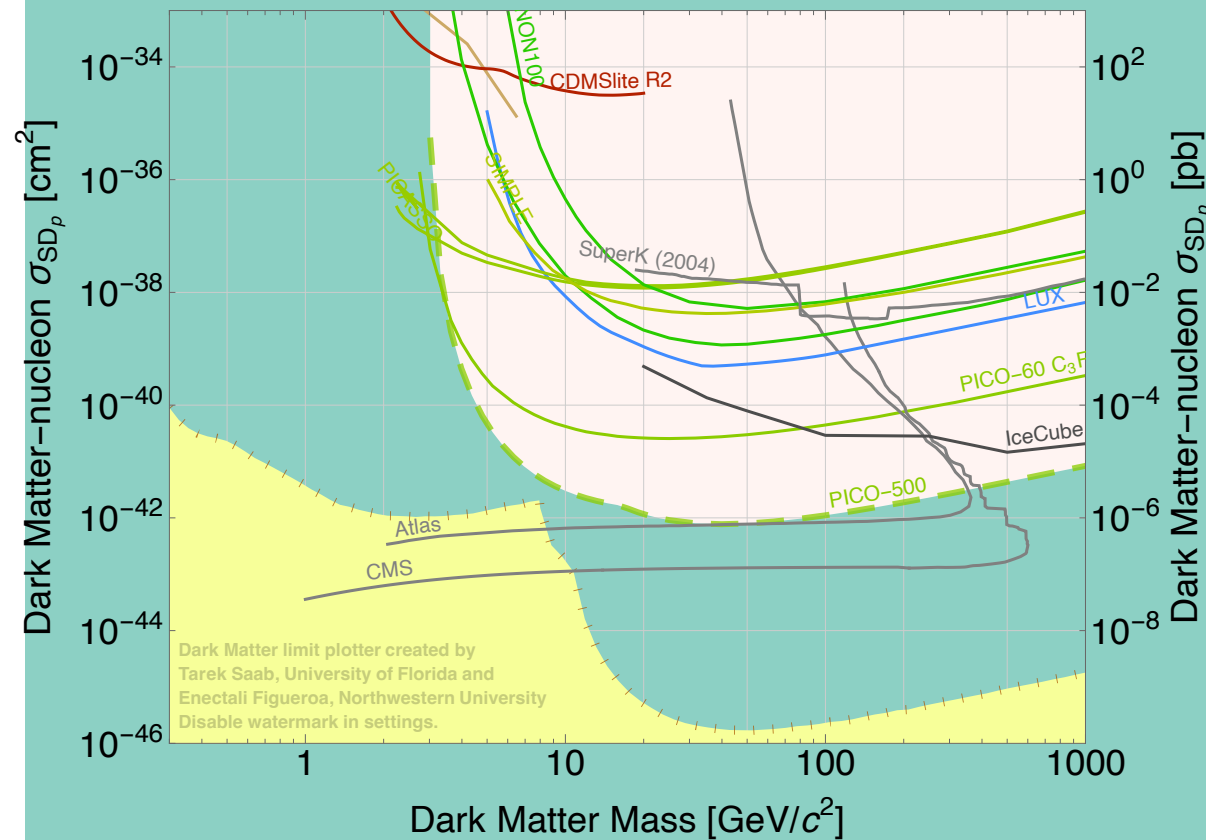
C. Amole et al. Dark Matter Search Results from the Complete Exposure of the PICO-60 C_3F_8 Bubble Chamber. *Phys. Rev. D*, 100(2):022001, 2019. doi: 10.1103/PhysRevD.100.022001.

BUBBLE CHAMBERS

- Fluorine is a particularly good target for searches for spin-dependent WIMP-nucleon interactions, and PICO produces leading limits here (model dependence determines if collider bounds are even better)
- Several nuclei can be used—can probe the nature of the WIMP-nucleon coupling
- Previous PICO generations (PICO-60) had the target hung under the bellows, PICO-40, in preparation for PICO-500 is “right side up”
- PICO-40L is commissioning, with a particular focus on background reduction, PICO-500 under construction

C. Amole et al. Dark Matter Search Results from the Complete Exposure of the PICO-60 C_3F_8 Bubble Chamber. Phys. Rev. D, 100(2):022001, 2019. doi: 10.1103/PhysRevD.100.022001.

Plot from Dark Matter Limit Plotter (Sep 12 2022)
<https://supercdms.slac.stanford.edu/dark-matter-limit-plotter> by Tarek Saab et al

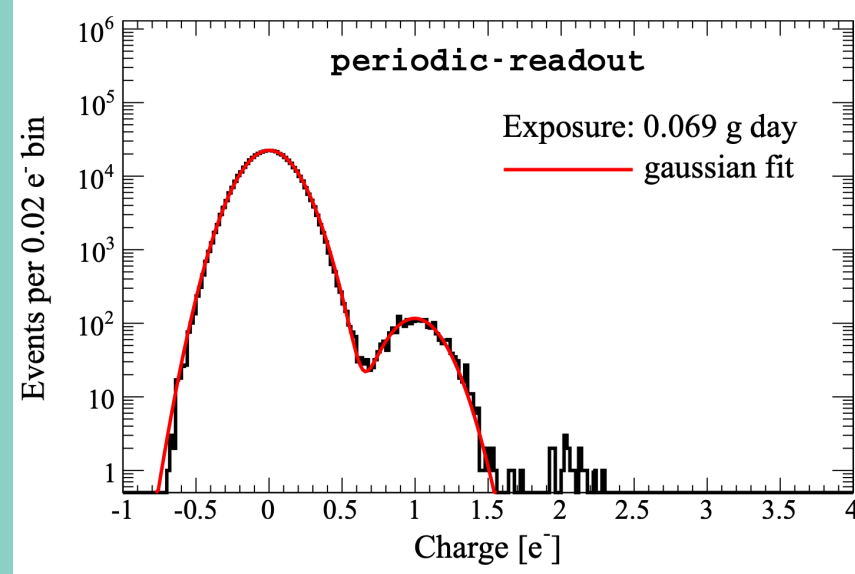


Caution: Combining collider and direct detection bounds like this involves some model dependence.

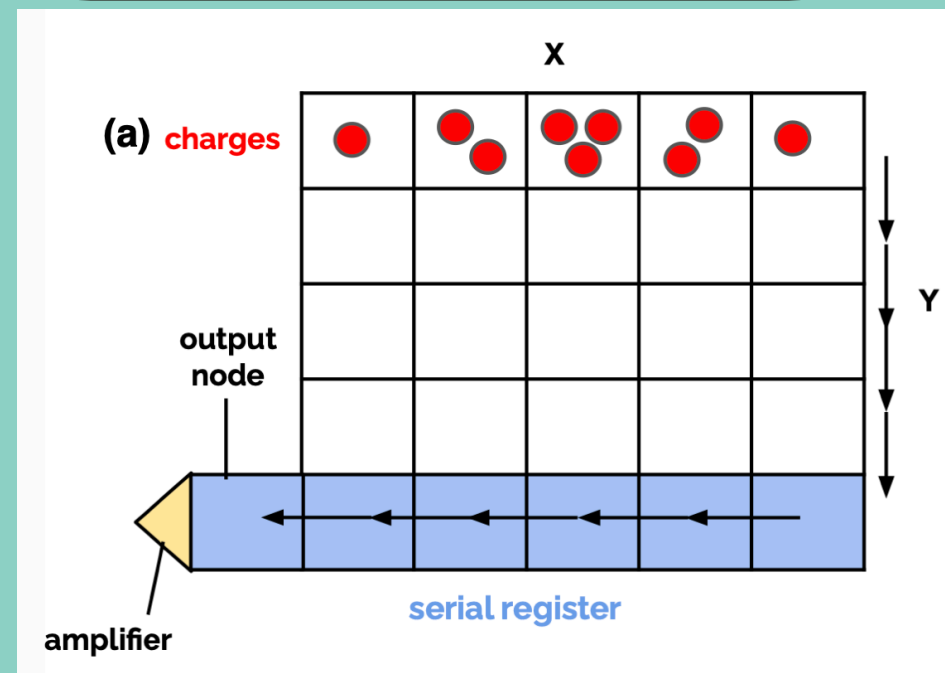


IONIZATION: CCDS

- DAMIC-M and SENSEI, using CCD sensors, have achieved energy thresholds of $\sim 50\text{eV}$
- Requires very low cosmogenic activation and Rn contamination— limit time aboveground
- Very low dark current achieved: $\sim 10^{-4}e^-/\text{pixel}/\text{day}$
- “Skipper” amplifiers read out the charge in a single pixel repeatedly, which gives single-electron resolution
- Scientific targets: DM-nucleon to $\sim 1\text{GeV}/c^2$ as well as DM- e^- scattering and full absorption (e.g. dark photon)



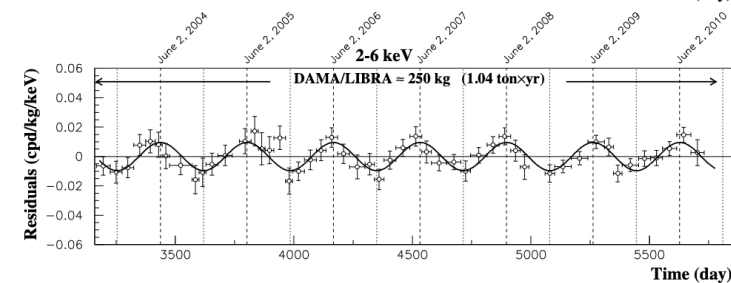
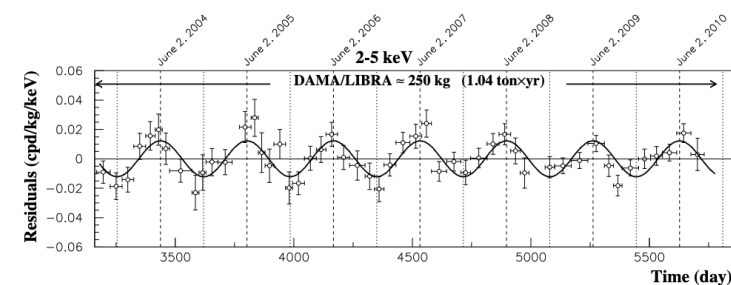
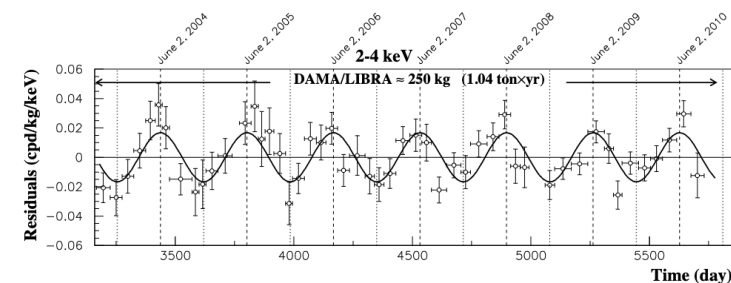
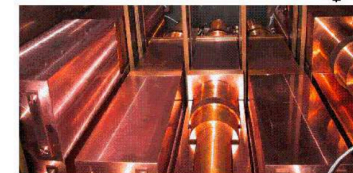
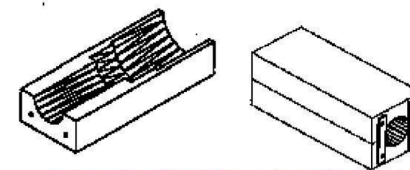
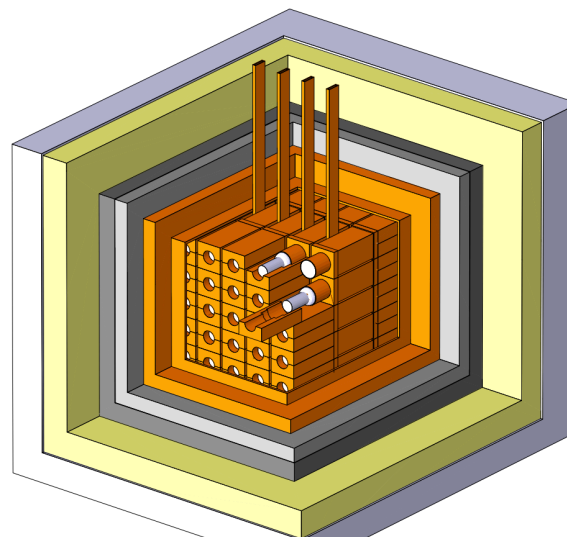
Orr Abramoff et al. SENSEI: Direct-Detection Constraints on Sub-GeV Dark Matter from a Shallow Underground Run Using a Prototype Skipper-CCD. Phys. Rev. Lett., 122(16):161801, 2019. doi: 10.1103/PhysRevLett.122.161801.



I. Arnquist et al. The DAMIC-M Experiment: Status and First Results. In 14th International Workshop on the Identification of Dark Matter 2022, 10 2022.

SCINTILLATION: DAMA

- Sodium iodide is a strong scintillator and the DAMA detectors observe 5 – 7 photoelectrons/keV, with an analysis threshold around a few keV
- Running for 14+ year, DAMA/Libra collected an exposure of a tonne-year
- And observed an excess, very high statistical significance for a modulation as expected from dark matter
- However, no other experiment has seen a similar excess, which is very challenging to combine with the strong signal this implies



The DAMA/LIBRA apparatus
<https://arxiv.org/pdf/0804.2738>
Final model independent result of DAMA/LIBRA-phase1
<https://arxiv.org/abs/1308.5109>

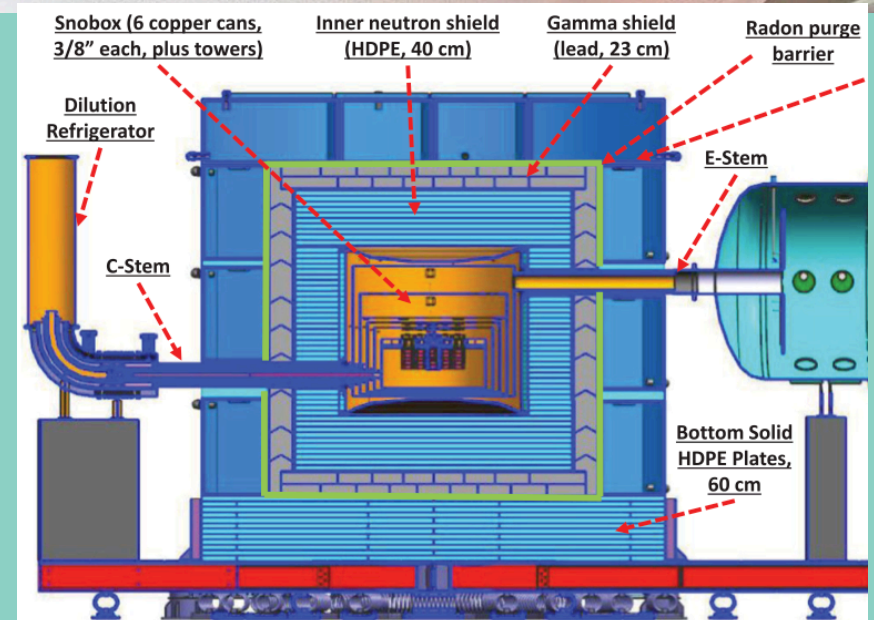
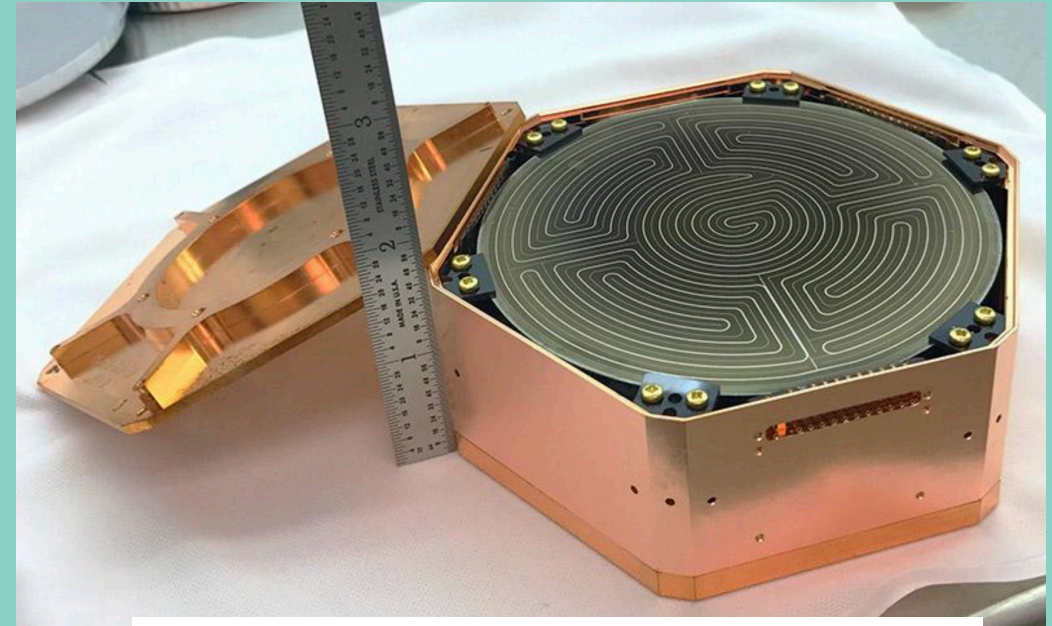
SUB-KELVIN DETECTORS

- **Light, $\sim \text{GeV}/c^2$ -scale and below dark matter scattering with target nuclei or electrons will deposit only some eV of energy**
- **The lowest-energy quanta available from scattering are phonons— vibration quanta of condensed materials**
- **Either by detecting a temperature increase in a target (thermal readout)**
- **Or sensing phonons from a scatter before thermalisation (athermal readout)**
- **these detectors can reach thresholds towards eV when operated at $\sim 10\text{mK}$**

CRYSTALS: CDMS

- SuperCDMS SNOLAB is the latest of a series of CDMS direct detection experiments
- Previously at the Soudan laboratory, but SNOLAB gives almost 3 times the overburden, and a factor 250 reduction in neutrons from muons.
- made up of solid-state detector modules of germanium (1.4kg) or silicon (0.6kg) crystals, held at 15 – 30mK
- Observables are heat (as phonons) and charge
- Phonons are read out with transition edge sensors (TES), sensitive to $\Delta T \sim 10\text{mK}$

Detector image courtesy of SuperCDMS
<https://supercdms.slac.stanford.edu/>

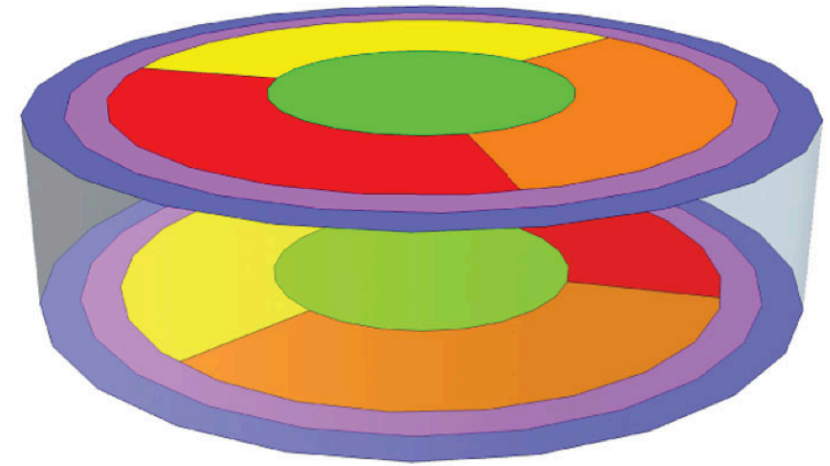


R. Agnese et al. Projected Sensitivity of the SuperCDMS SNOLAB experiment. Phys. Rev. D, 95(8):082002, 2017. doi: 10.1103/PhysRevD.95.082002.

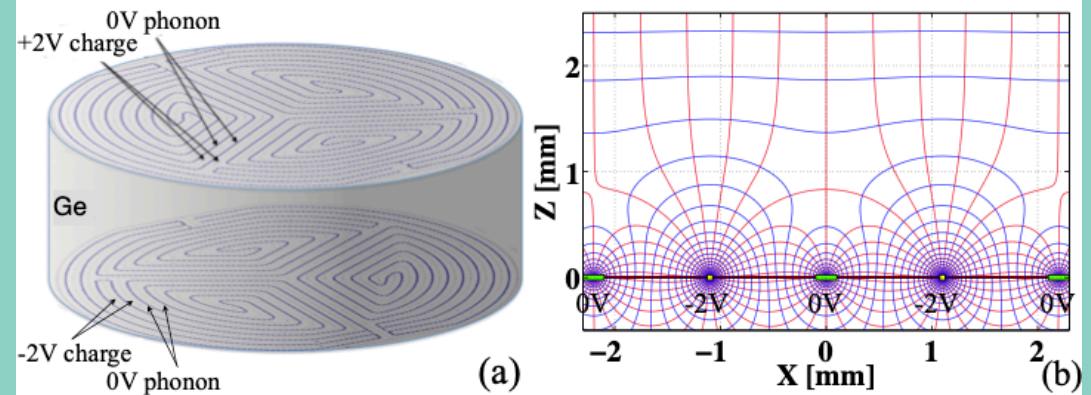
CRYSTALS: CDMS

- SuperCDMS uses two run modes:
- The CDMSlite mode uses a high amplification voltage to lower the energy threshold
- Under a higher drift field, the charge carriers drifted through the detector emit additional phonons, which are then read out
- Downside: lower ER/NR discrimination capability
- The interleaved mode (iZIP) reads out charge as well as phonons
- Rejects ER events based on charge/phonon ratio
- Surface backgrounds are also reduced with a positive bias on the ionisation channels, funnelling the charges

HV:

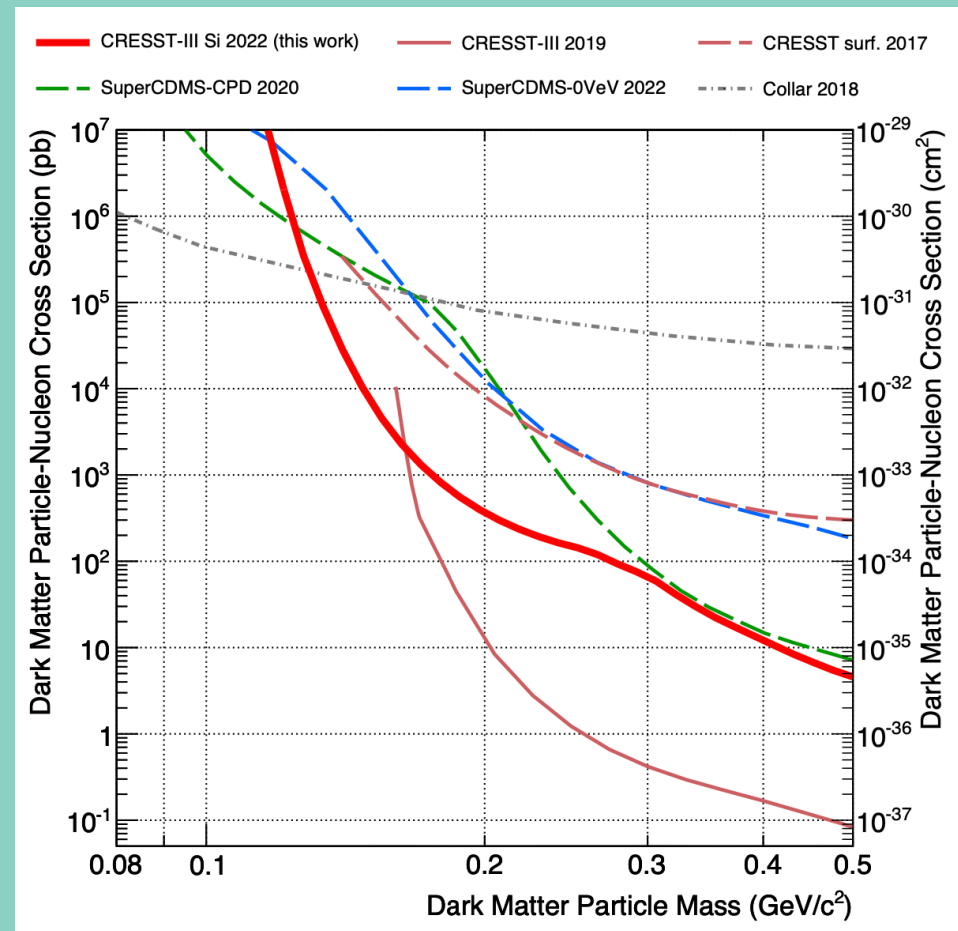
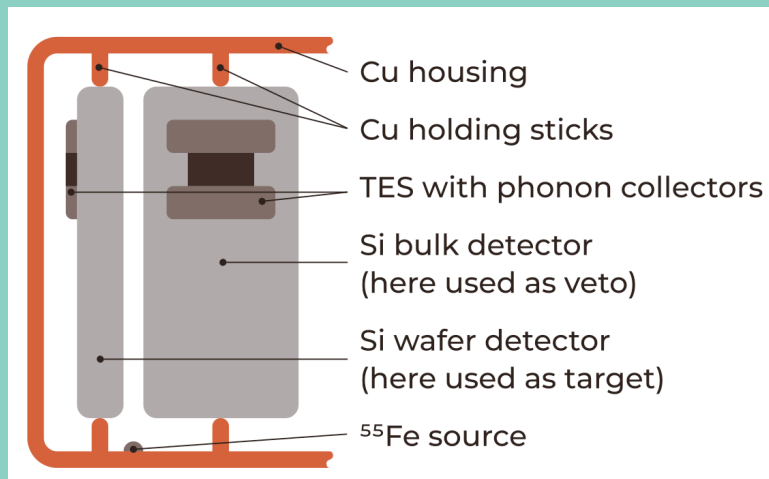


Phonon channel segmentation shown with color

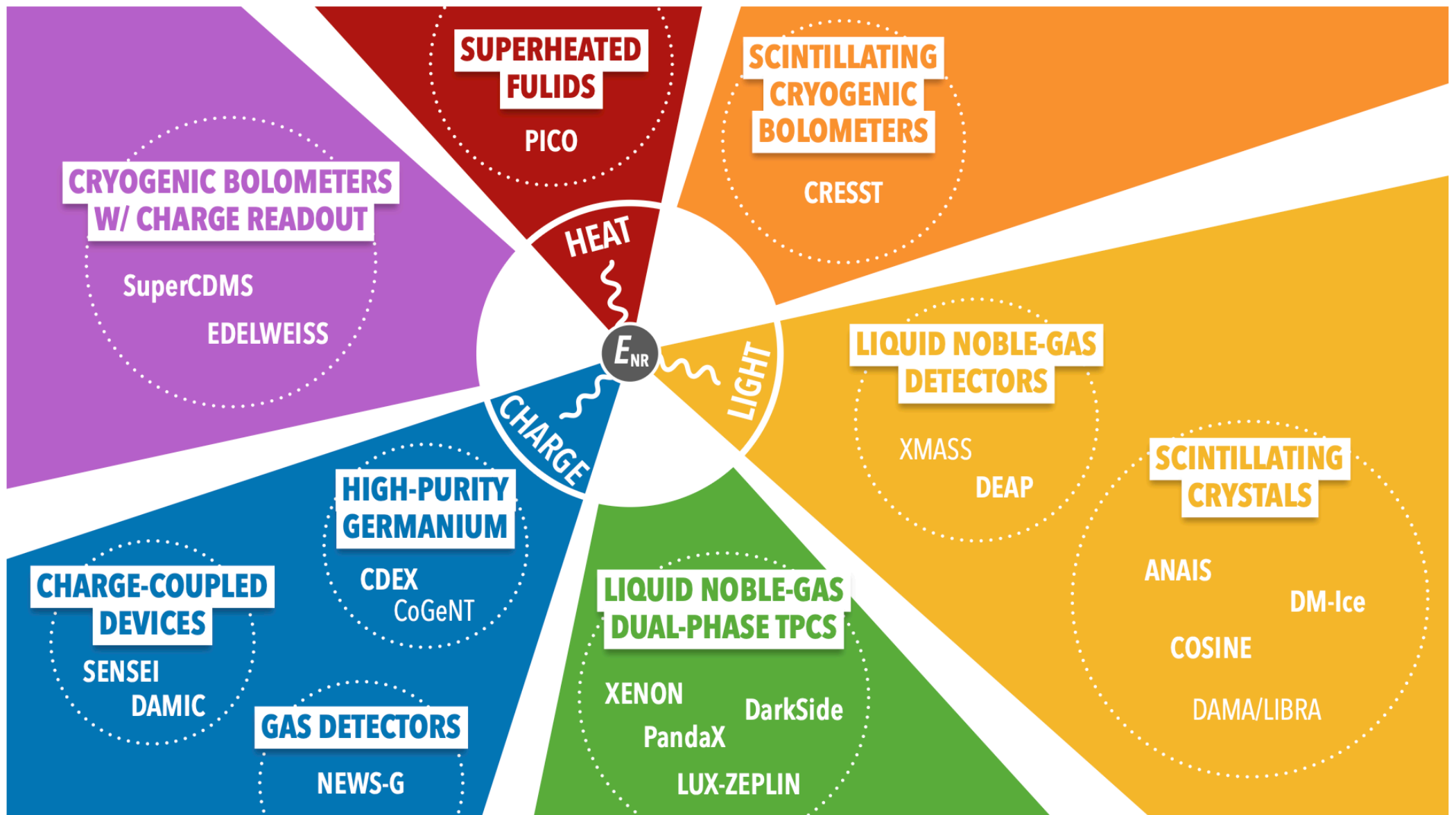


CRYSTALS: CRESST-III

- **A range of crystal targets; CaWO_4 , Al_2O_3 , LiAlO_2 , Si held at 10 mK, and operating as calorimeters read out by a transition-edge sensor**
- **Observables: phonons and (for scintillators) scintillation light, which provides ER/NR discrimination**
- **Choosing a specific low-noise crystal, CRESST-III was able to put limits down to $0.115 \text{ GeV}/c^2$ with an energy threshold of 10 eV**



SIGNATURES OF A DARK MATTER RECOIL



STATISTICAL INTERPRETATION OF RARE EVENT SEARCHES

AUTHORITY

Whatever superstitions the sperm whalemens in general have connected with the sight of this object, certain it is, that a glimpse of it being so very unusual, that circumstance has gone far to invest it with portentousness. So rarely is it beheld, that though one and all of them declare it to be the largest animated thing in the ocean, yet very few of them have any but the most vague ideas concerning its true nature and form; notwithstanding, they believe it to furnish to the sperm whale his only food. For though other species of whales find their food above water, and may be seen by man in the act of feeding, the spermaceti whale obtains his whole food in unknown zones below the surface; and **only by inference is it that any one can tell** of what, precisely, that food consists. At times, when closely pursued, he will disgorge what are supposed to be the detached arms of the squid; some of them thus exhibited exceeding twenty and thirty feet in length. They fancy that the monster to which these arms belonged ordinarily clings by them to the bed of the ocean; and that the sperm whale, unlike other species, is supplied with teeth in order to attack and tear it.

OVERVIEW

- **In order to have the required background for the hands-on session,**
- **and to prepare you for thinking about the details of analysis in the coming lectures**
- **I'll spend one of these sessions on the statistical interpretation of rare event searches.**
- **The low number of events,**
- **high stakes of discovering a new fundamental particle**
- **oftentimes uncertain background shapes**
- **etc. means that care is required to draw conclusions from the data you see in your direct detection experiment**

NOTATION

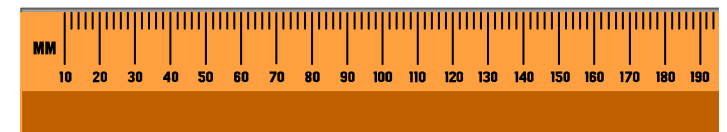
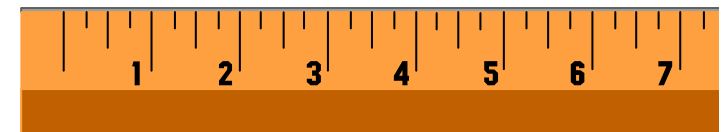
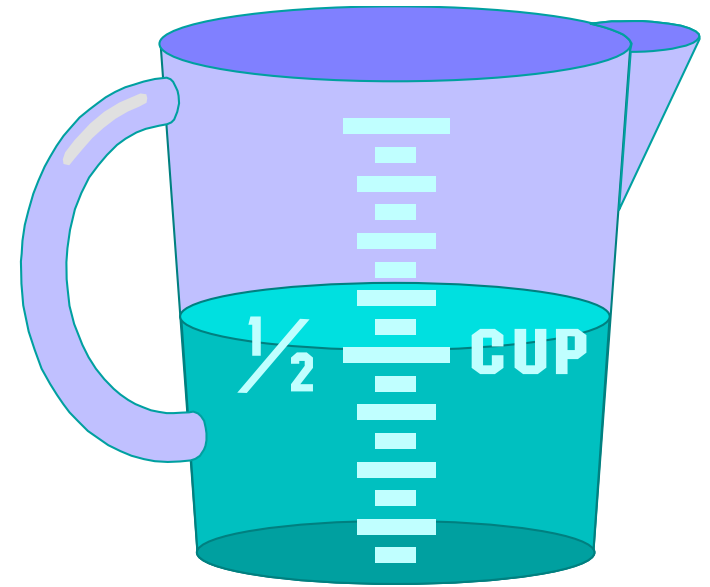
I'll follow Frederick James' Statistical Methods

- **A random variable, or several are X, X_i, \mathbf{X}**
- **The probability of an event A is $P(A)$**
- **Parameters of a model are θ**
- **Conditional probabilities are $P(A | B)$**
- **The likelihood is $\mathcal{L}(\theta | X) = P(X | \theta)$**
- **Expectation value(s) for counting experiments are μ, μ**
- **Expectation values, variance $E(X), V(X)$**
- **best-fit parameters or point estimates are $\hat{\theta}$**

OBSERVED DATA IS RANDOM VARIABLES

or is it are

- Our measured data is a result of processes both truly and practically random (e.g. quantum processes, me reading a ruler crooked)
- In some cases, the data itself is close to what we wish to measure, and we hardly think of ourselves doing statistics
- However, in particular when looking for small or subtle effects, the random noise may be significant, and the relationship between physics parameters and the measured quantity less straightforward
 - You'll need to make a statistical model for how your data came to be,
 - And methods to make sound conclusions

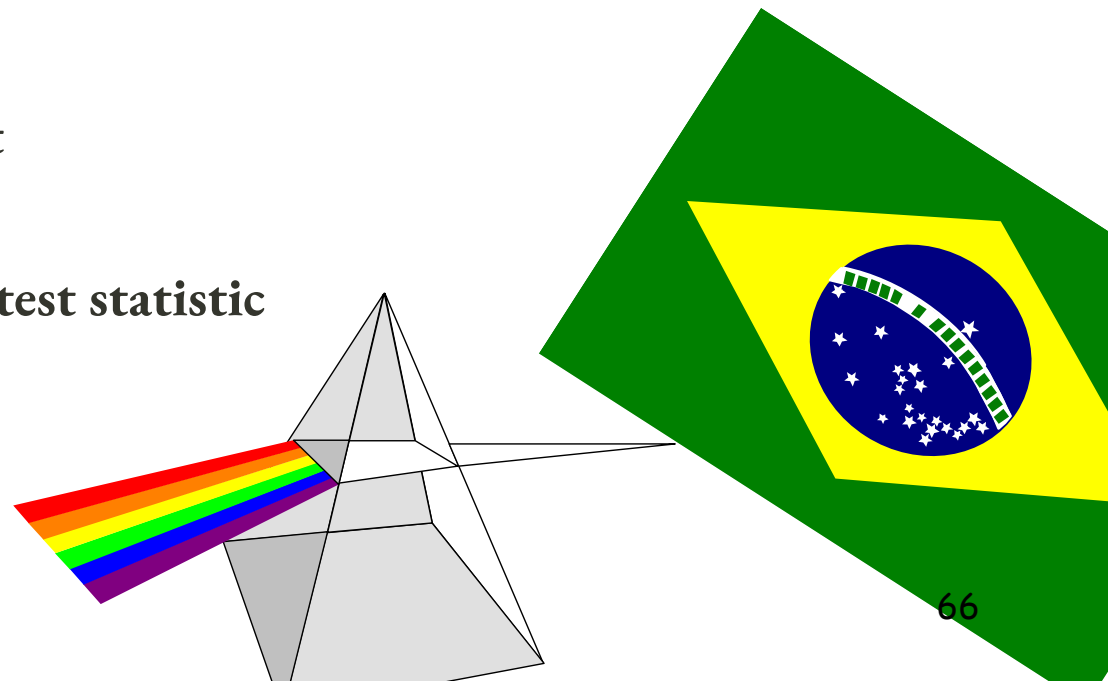


TEST STATISTICS ARE FUNCTIONS OF YOUR OBSERVATIONS

- Any function of your observed data will be a random variable
- By using the right function, we can gather all the information gathered into one number
 - E.g. estimators (\hat{s}) which directly give a measurement of some parameter
- The tricky part will most often be to
 - choose the function to give the most information from the data, and
 - Understand the *distribution* of the test statistic

$$\hat{\mu} = \frac{1}{N} \sum_i x_i$$

$$\hat{\sigma} = \sqrt{\frac{\sum_i (x_i - \hat{\mu})^2}{N - 1}}$$



PROBABILITY DISTRIBUTIONS/DENSITIES

- if X is a continuous variable, we may define a probability *density* function (PDF) to describe the distribution
- The cumulative density function (CDF), $F(X)$, is often also useful
 - and its inverse!

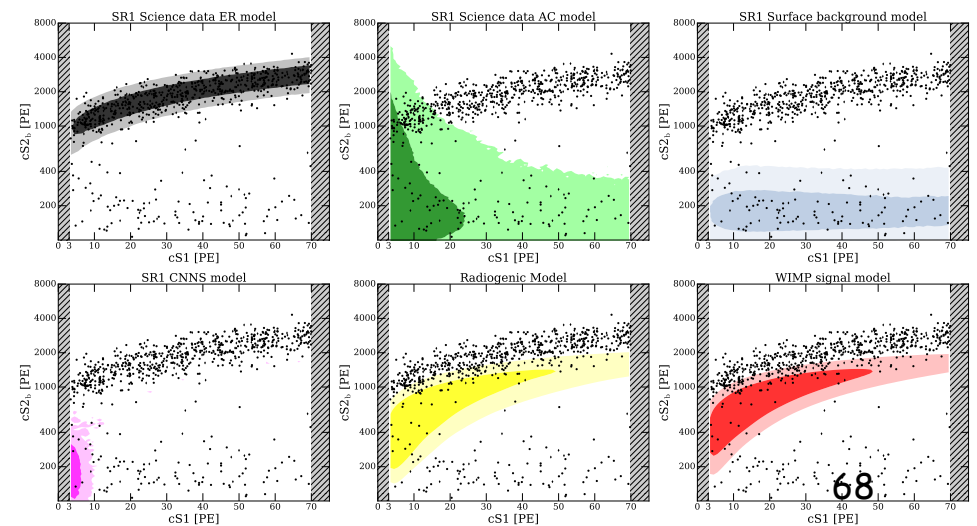
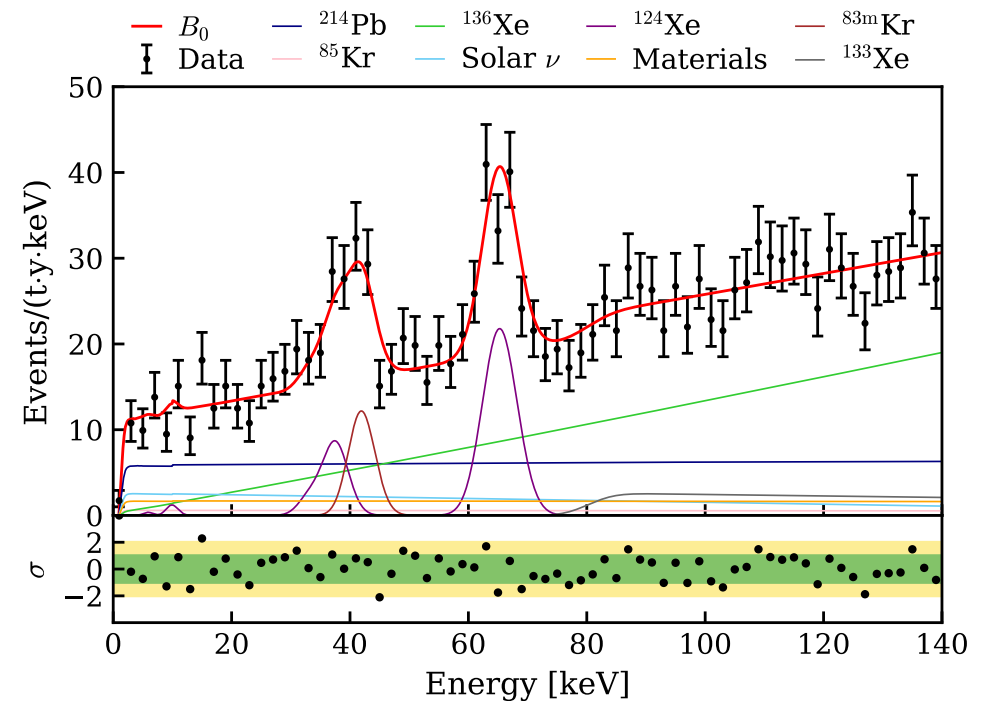
$$f(X) = \lim_{\epsilon \rightarrow 0} P(x_0 < X < x_0 + \epsilon) / \epsilon$$

$$F(X) = \int_{-\infty}^X f(X') dX'$$

$$P(X_0 < X < X_1) = F(X_1) - F(X_0)$$

ANY NUMBER OF DISTRIBUTIONS!

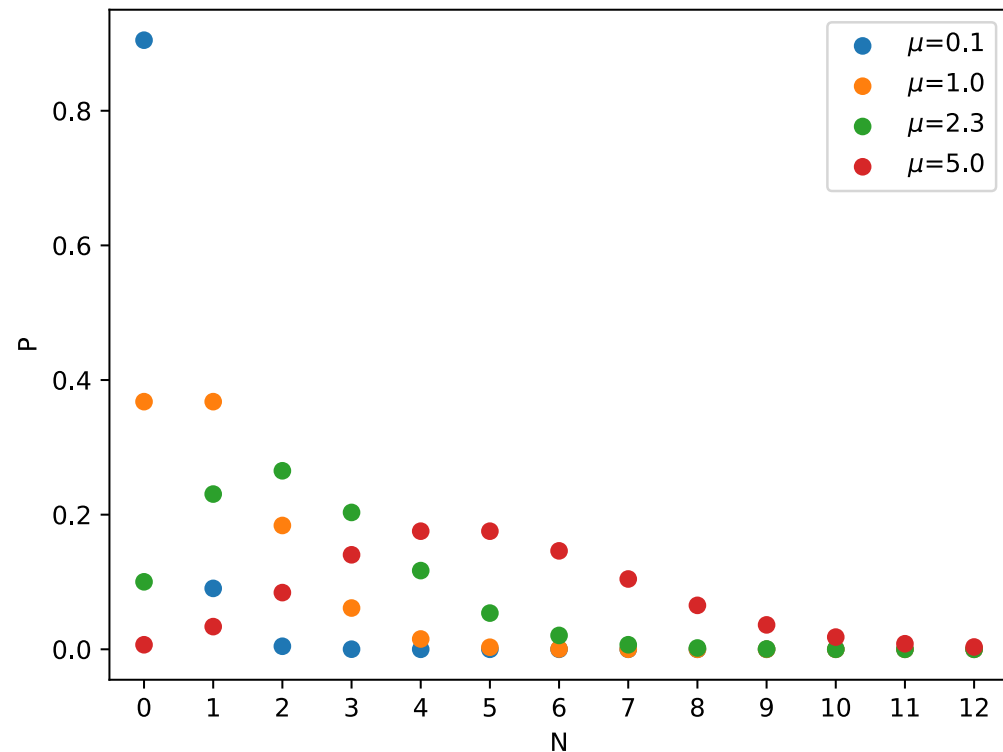
- If we are certain about the outcome, is it really an experiment?
- Depending on what you measure, your distributions may be as simple or as complicated as can be imagined
- However, for many problems, physical considerations or your experience may lead you to have a look at some of the most common ones used— they are useful building blocks!
- Some (student T, F-test, χ^2) are also useful because they describe the behaviour of some useful test statistics



THE POISSON DISTRIBUTION

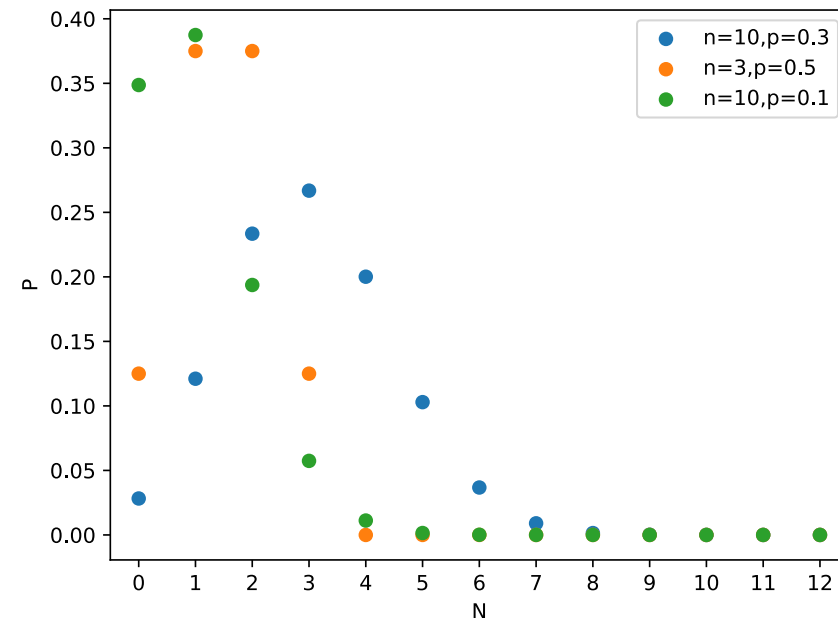
$$P(N) = \frac{\mu^N e^{-\mu}}{N!}$$

- If you count events that happen in a certain period, you'll end up with a Poisson distribution
- Expectation value and variance are both μ



BI/MULTI-NOMIAL DISTRIBUTIONS

- If we count how many times each of a finite set of outcomes happens, we get the multinomial distribution
- M total tries, n_i events in each category, with probability p_i
- And if the number of possible outcomes $k = 2$, we get the Binomial distribution
- Examples: Histogram bin counts, classification

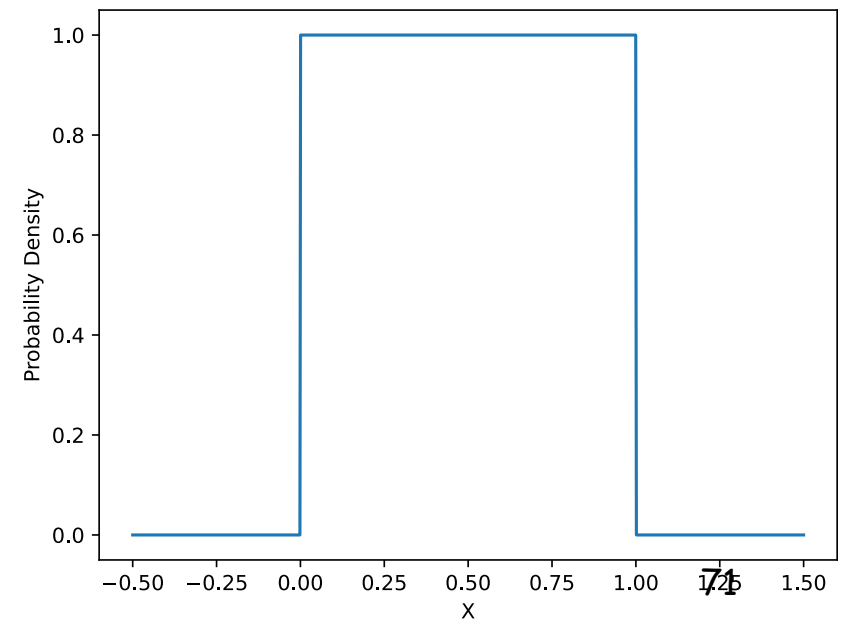


UNIFORM DISTRIBUTIONS

- Turns up in e.g.
- Spatial distribution of dark matter events?
- But more importantly, it is often very often useful to convert another distribution into a uniform distribution (Y here) between 0 and 1

$$Y(X) = \int_{-\infty}^X f(X') dX'$$

$$f(X) = \begin{cases} \frac{1}{b-a} & \text{if } a < X < b \\ 0 & \text{else} \end{cases}$$

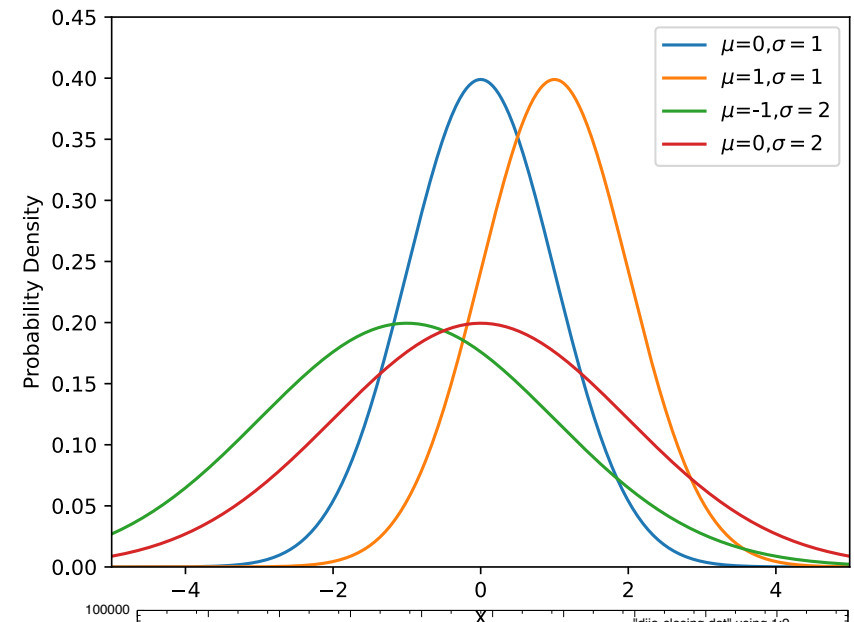


THE GAUSSIAN DISTRIBUTION

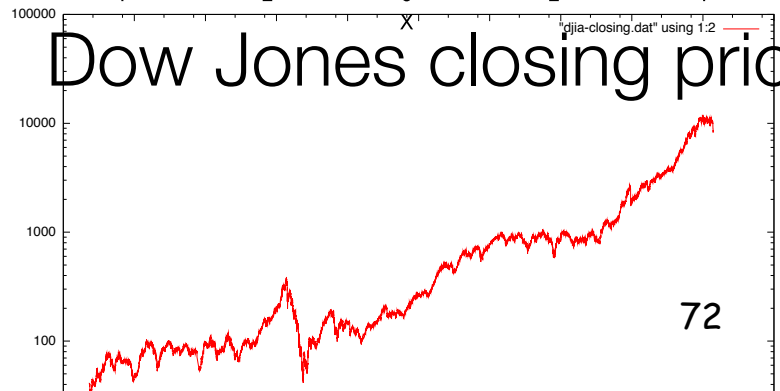
The industry default. AKA bell curve, normal distribution

- **The Gaussian distribution is the limit of sums of random numbers with finite mean and variance— the Central Limit Theorem**
- **E.g. — diffusion!**
- **For this reason, it is often the “default” assumption for a continuous distribution**
- **However, by using this (or many other analytical distributions) you may be assuming to know the behaviour for even very extreme outliers**

$$f(X) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-(x-\mu)^2/(2\sigma^2)}$$



Dow Jones closing price

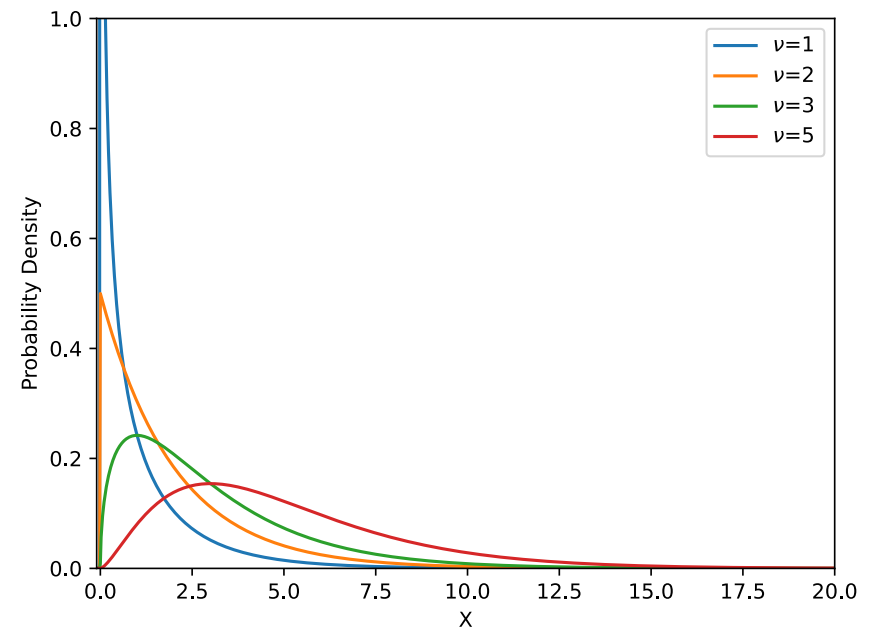


THE χ^2 -DISTRIBUTION

- The sum of the square of ν standard normal distributed numbers is distributed according to the χ^2 -distribution
- We'll see later that this means that you'll encounter this distribution frequently when computing confidence intervals

$$\sum_{i=1}^N \frac{(X_i - \mu_i)^2}{\sigma_i^2} \sim \chi_{\nu=N}^2$$

$$f(X | \nu) = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} X^{\nu/2-1} e^{-x/2}$$



HISTOGRAMS AS DISTRIBUTION ESTIMATES

- If you wish to characterise the distribution of, for example, the distribution of energy deposited by electrons and photons in a calorimeter, or the total path length of all tracks, you may never find an analytical estimate
- Higher dimensionality can challenge this approach
- and you'll need to check you have enough samples or include the uncertainty

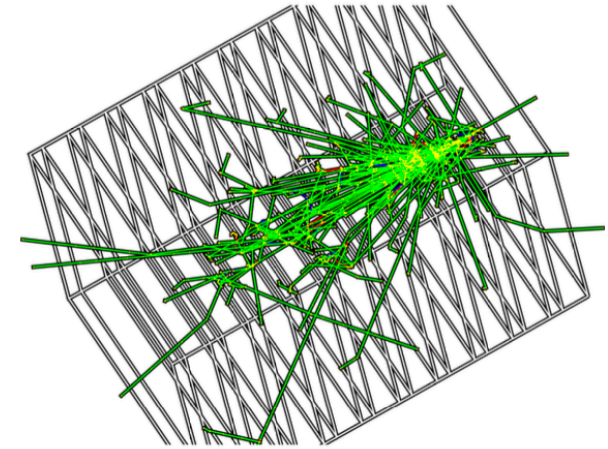
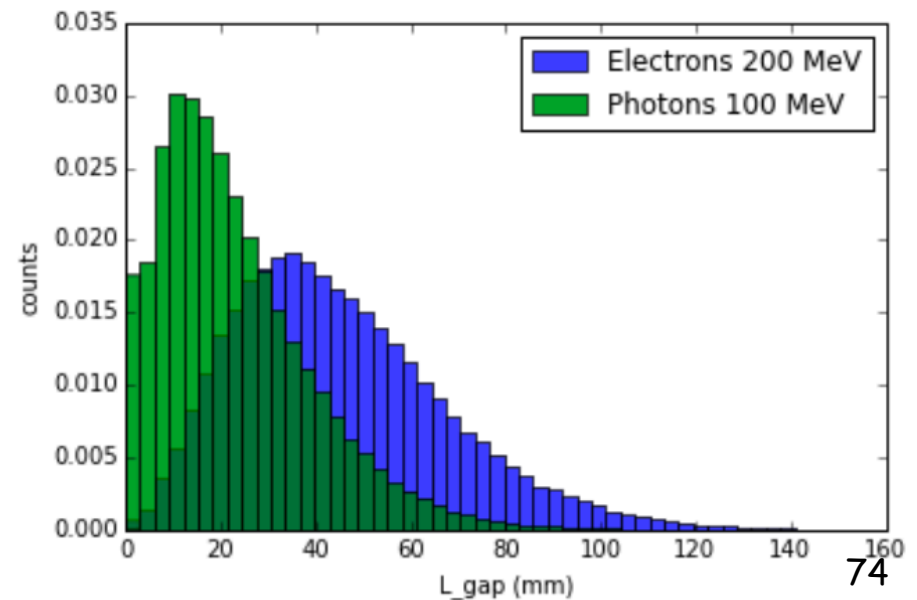


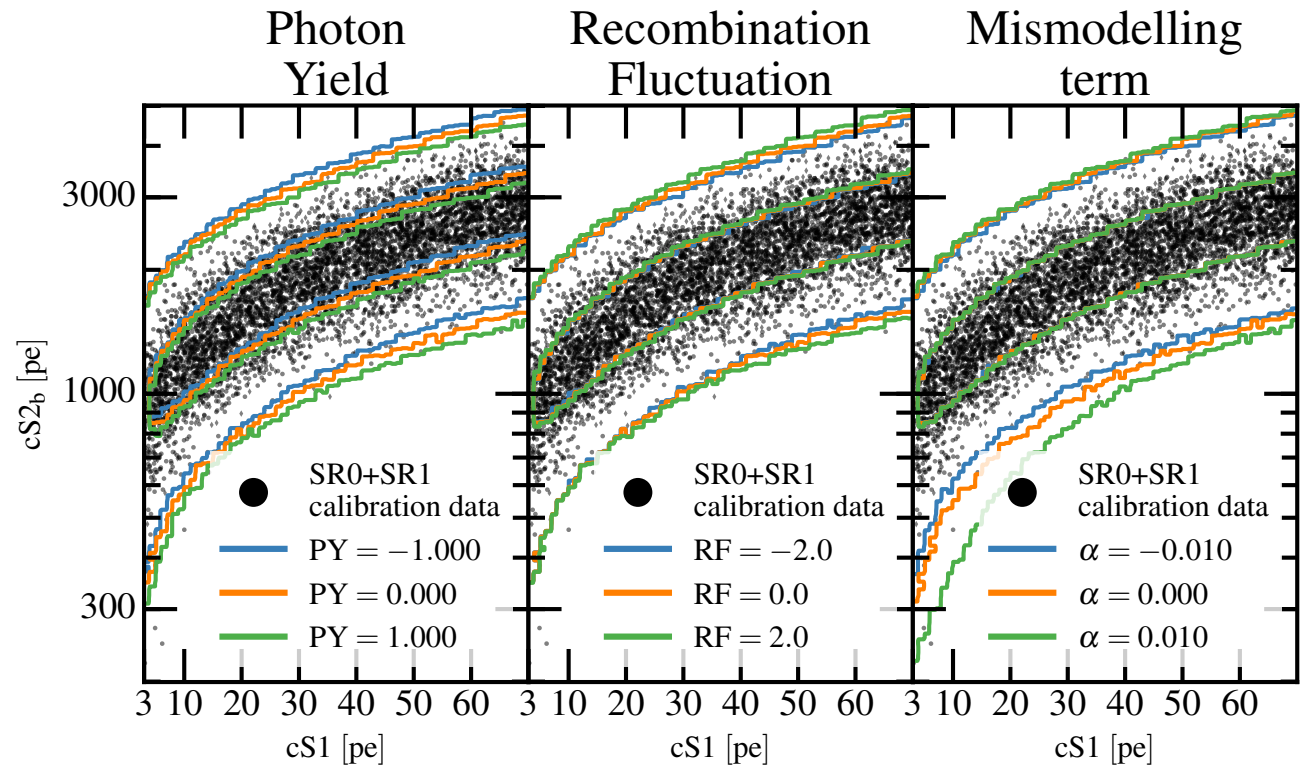
Figure 1: Electromagnetic shower in calorimeter induced by photon



Fitting using finite Monte Carlo samples (Barlow and Beeston)

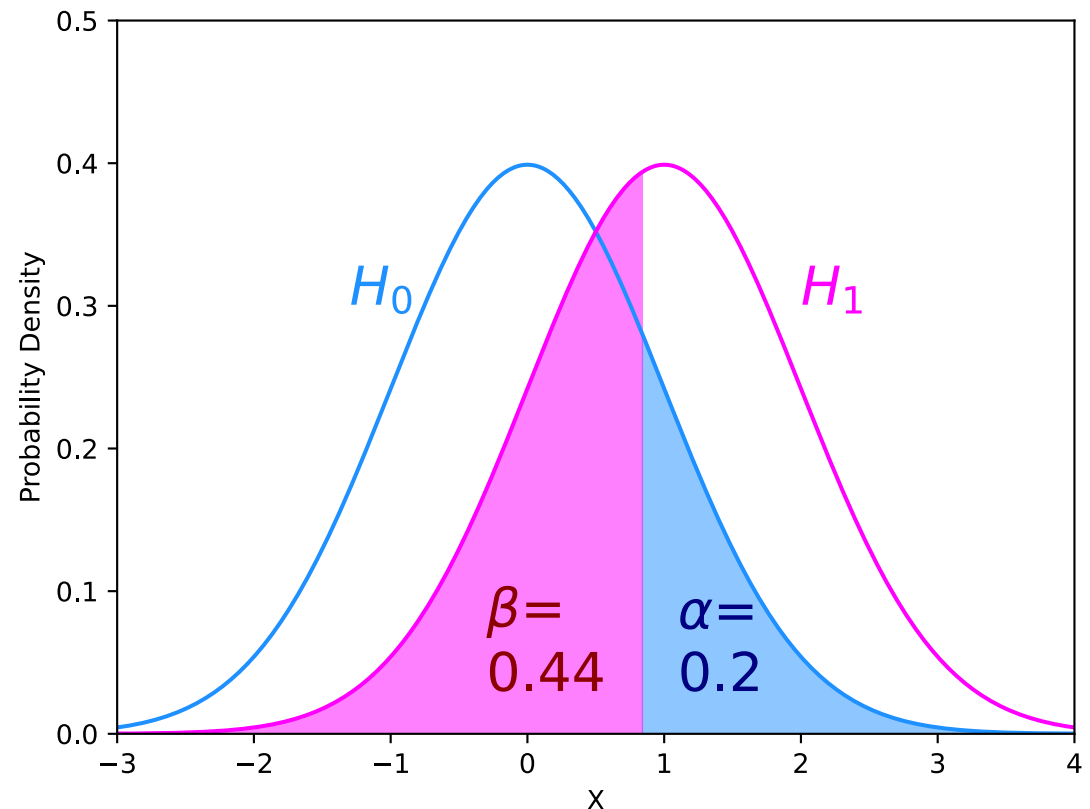
TECHNICAL ASIDE 1: SHAPE AND RATE?

- When using histograms to estimate the distribution, nuisance parameters are well-named
- To have a continuous nuisance parameter, “template morphing” -- linear interpolation between some points in parameter space is often used
- Since this is computationally tricky, there will often be a divide between “rate parameters” -- those that only affect expectation values, and therefore are “easy” and “shape parameters” -- those that require modifying the PDF of one or more signal/background model



HYPOTHESIS TESTING

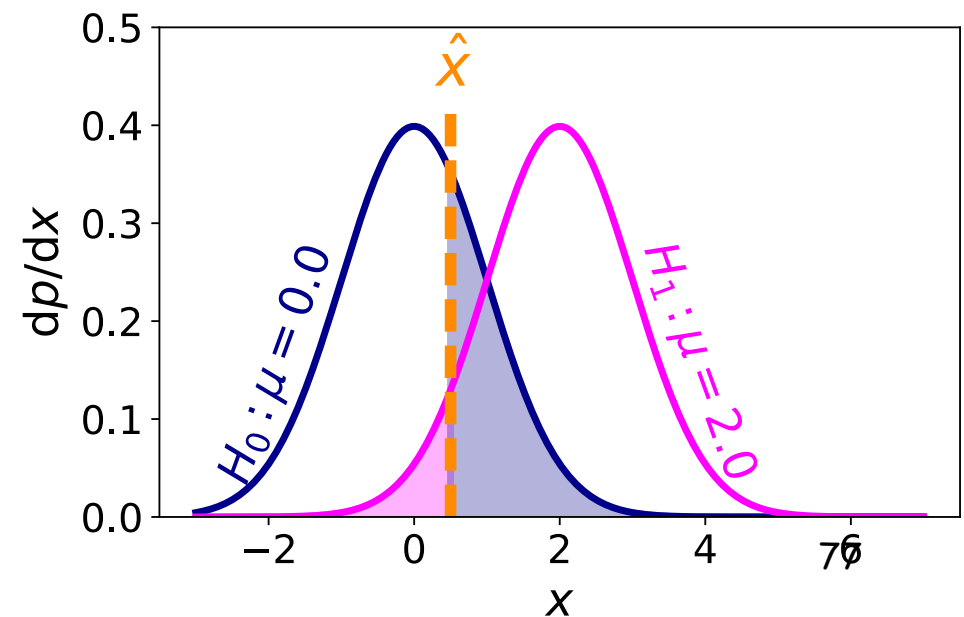
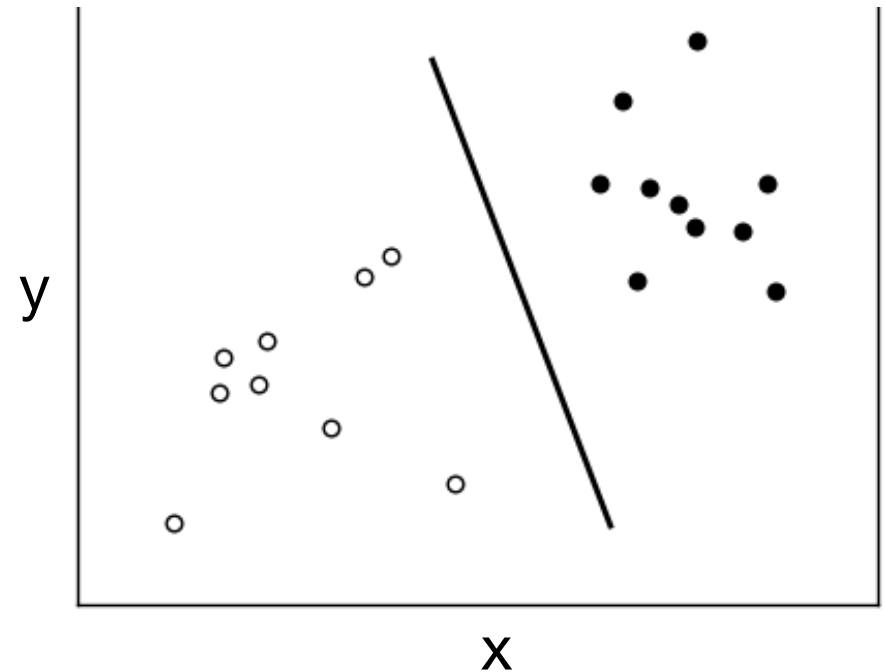
- **Frequentist hypothesis testing: make a decision between the two alternatives**
- **You get to choose:**
 - **What test statistic you use to separate the two hypotheses!**
 - **And, the decision boundary, either explicitly**
 - **Or implicitly by demanding a certain probability to reject H_0**



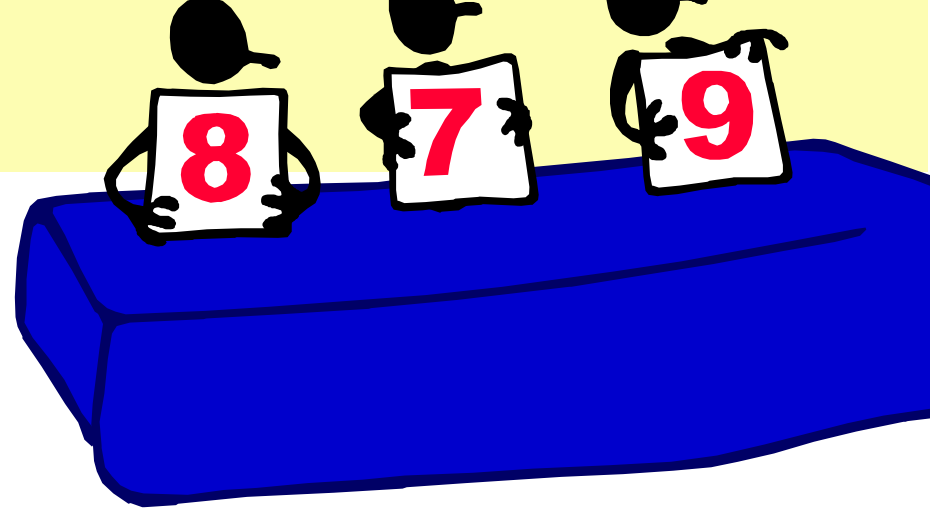
	P(accept H_0)	P(accept H_1)
H_0 is true	$1-\alpha$	α (test size)
H_1 is true	β	$1-\beta$ (power)

TEST STATISTICS

- From the collected data, we wish to find a function of the data that expresses a direction or ordering of the data in a more H_0 or H_1 direction
- Typical examples; mean, median etc.
- For the example to the right, y would be a poor test statistic if we wish to distinguish the two, x would be better, and a combination would provide very good separation



WHAT IS A P-VALUE?



- Since we want to use the best test statistic for each case, we could have many ways of measuring agreement with a hypothesis
- However, we can transform all our rulers into the same space by using p-values, which works with the integral of the distribution of T
- all p-values are between 0 and 1, and are defined by deciding on:
 - a test statistic
 - and a decision of what direction that test statistic expresses more tension with H_0
- Under H_0 , p is uniformly distributed between 0 and 1

$$p(T_{\text{obs}}) = \int_{T_{\text{obs}}}^{\infty} f(T | H_0) dT$$

P-VALUES ARE THE PROBABILITY TO OBSERVE A DATASET EQUALLY OR MORE EXTREME* THAN THE ONE OBSERVED, GIVEN A CERTAIN (NULL) HYPOTHESIS

ordering by a test statistic*

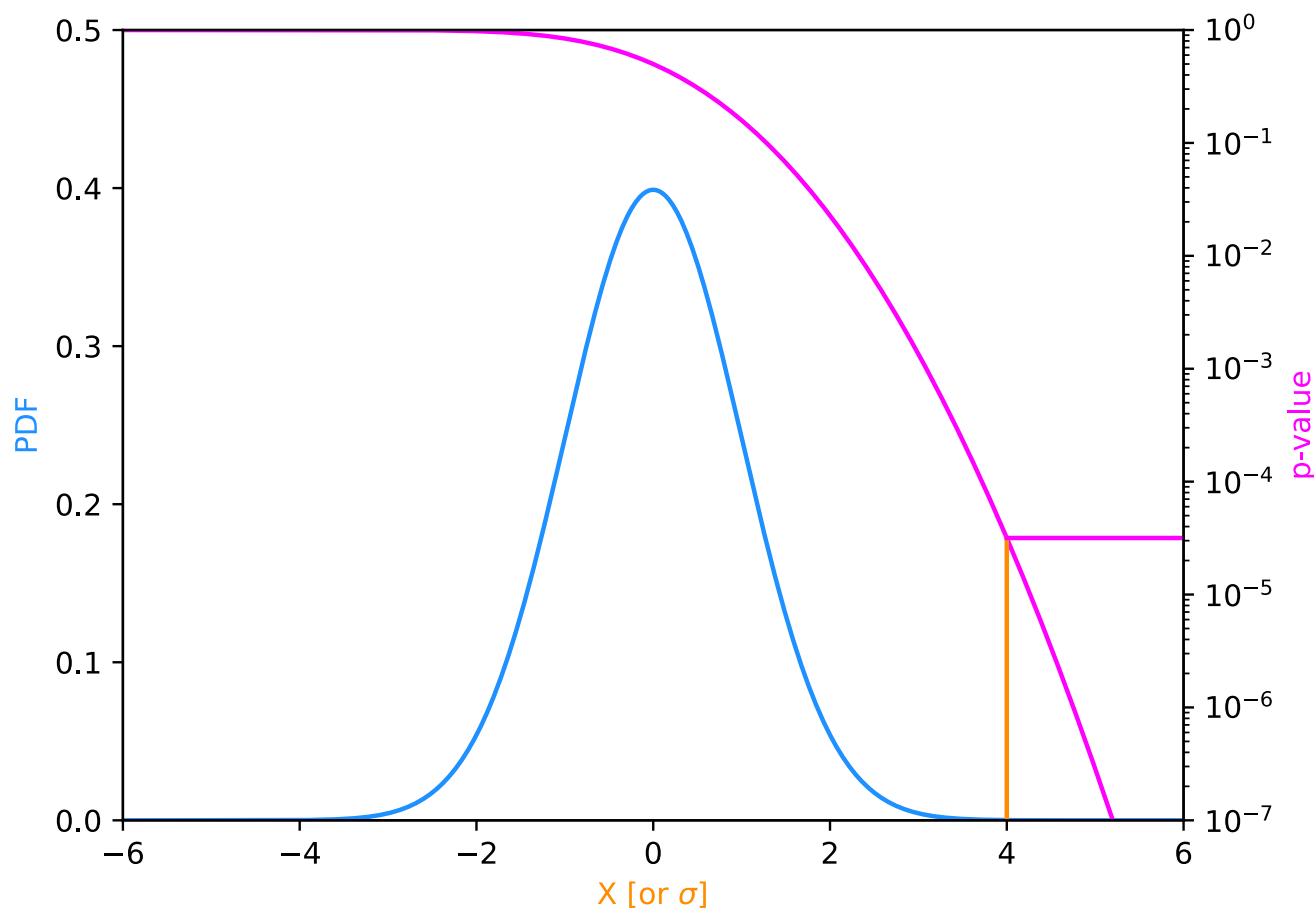
****usually chosen to separate the null and alternative hypothesis as well as possible**

“COUNTING SIGMAS”

improved analyses of the $H \rightarrow ZZ^{(*)} \rightarrow 4\ell$ and $H \rightarrow \gamma\gamma$ channels in the 7 TeV data. Clear evidence for the production of a neutral boson with a measured mass of 126.0 ± 0.4 (stat) ± 0.4 (sys) GeV is presented. This observation, which has a significance of 5.9 standard deviations, corresponding to a background fluctuation probability of 1.7×10^{-9} , is compatible with the production and decay of the Standard Model Higgs boson.

$$\sigma = \Phi^{-1}(1 - p)$$

- As a yardstick for p-values, you can often see “sigmas”, or σ (or Z-score) used.
- “Five sigma”, or 3×10^{-7} is the “standard” for discovery
- Though you should consider what is the appropriate threshold in your field
- Be wary that you often also see the 2-sided version!



THE LIKELIHOOD

- A very useful test statistic is likelihoods— the probability of the data *given* a model
- Likelihoods are central to most of both Bayesian and Frequentist methods
- As an example, the likelihood as a function of expected events for a counting experiment that sees 3 events is:
- We often deal with independent events (e.g. number of events in different histogram bins); we can build up a total likelihood by multiplying (or, using logarithms, adding) terms
- The well-loved χ^2 -statistic is what you get if you combine Gaussian likelihood terms

$$\mathcal{L} = P(\text{data}|H)$$

$$\mathcal{L}(\mu|N = 3) = \text{Poisson}(N = 3|\mu)$$

$$\mathcal{L}(\vec{\mu}|\vec{N}) = \prod_i \text{Poisson}(N_i|\mu_i)$$

$$\log(\mathcal{L}(\vec{\mu}|\vec{x}, \vec{\sigma})) =$$

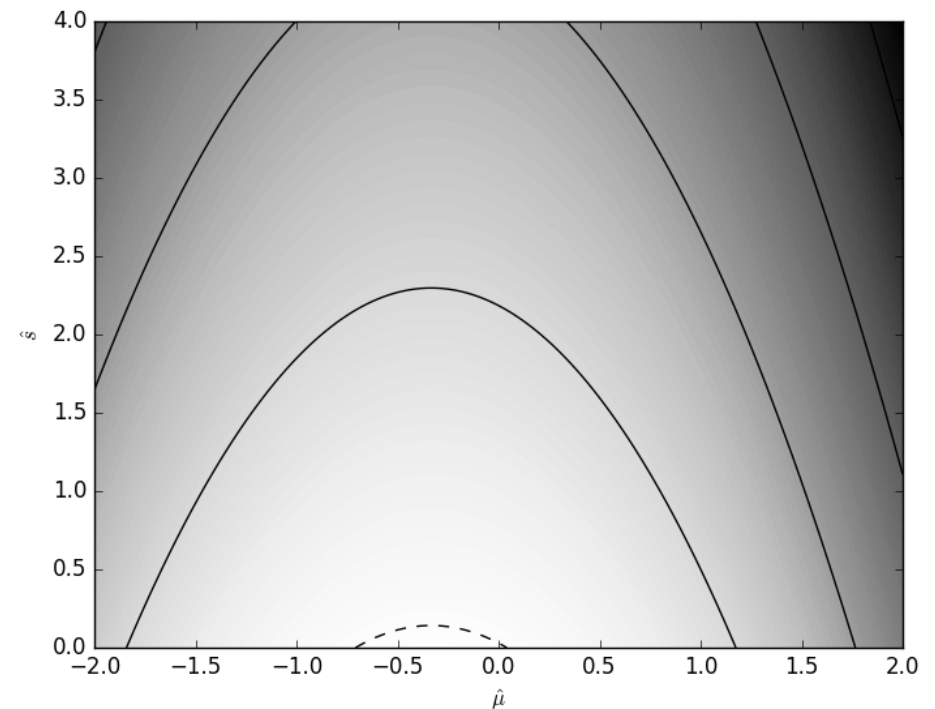
$$\sum_i \log(\text{Gaussian}(x_i|\mu_i, \sigma_i)) =$$

$$\sum_i \left(\frac{(x_i - \mu_i)^2}{\sigma_i^2} \right) + K$$

THE NEYMAN-PEARSON LEMMA

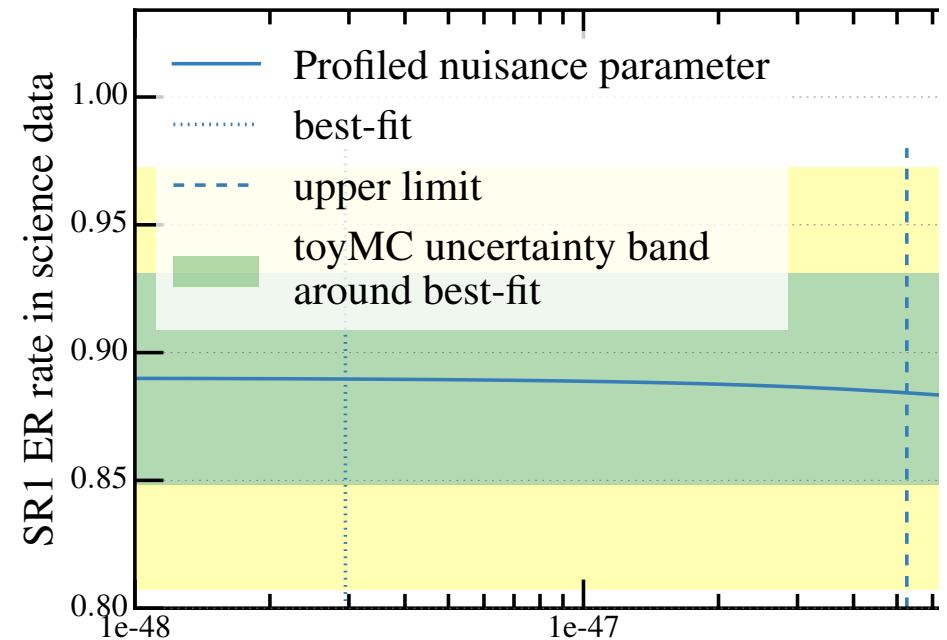
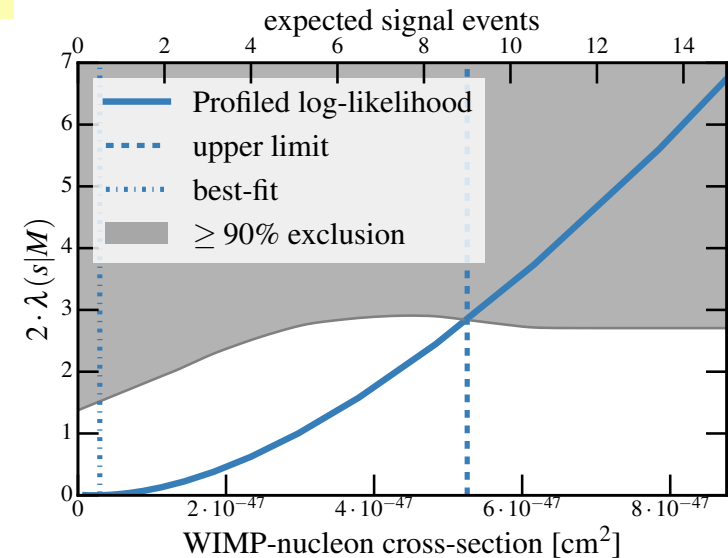
- IFF H_0 and H_1 are completely specified, the likelihood ratio between the two turns out to be the solution to the test statistic problem—it is the *uniformly most powerful test*.
- For example, the plot to the right shows the NP ratio between two Gaussian hypotheses, one with $\mu, \sigma = 0, 1$ and one 1, 2.

$$\lambda = \frac{\mathcal{L}(H_1)}{\mathcal{L}(H_0)}$$



PROFILING LIKELIHOODS

- We seldom have completely specified hypotheses
- Our background and signal models have uncertainties, parameterised by nuisance parameters (theta below)
- Unlike the Neyman-Pearson case, we are not guaranteed that this is the best possible test, but it very often performs well.



$$\lambda(s) = -2 \cdot (\log(\mathcal{L}(s, \hat{\theta})) - \log(\mathcal{L}(\hat{s}, \hat{\theta})))$$

FROM THE EARLIEST DAYS OF STATISTICS, STATISTICIANS HAVE BEGUN THEIR ANALYSIS BY PROPOSING A DISTRIBUTION FOR THEIR OBSERVATIONS, AND THEN, PERHAPS WITH SOMEWHAT LESS ENTHUSIASM, HAVE CHECKED WHETHER THIS DISTRIBUTION IS TRUE

- RALPH B. D'ANGOSTINO AND MICHAEL A. STEPHENS, *GOODNESS-OF-FIT TECHNIQUES*, 1986

GOODNESS-OF-FIT (GOF)

- The conclusions we draw from our data depends on our statistical model
- Unless we have a strong physical argument for a certain distribution to hold (e.g. Poisson for counting events) we should probe the correctness of our model or fit to the data
- Unlike other hypothesis testing, GOF tests must consider every possible other alternative as a competitor to the model we test
- The conclusion to a failed goodness-of-fit test may therefore sometimes just be “worry more”



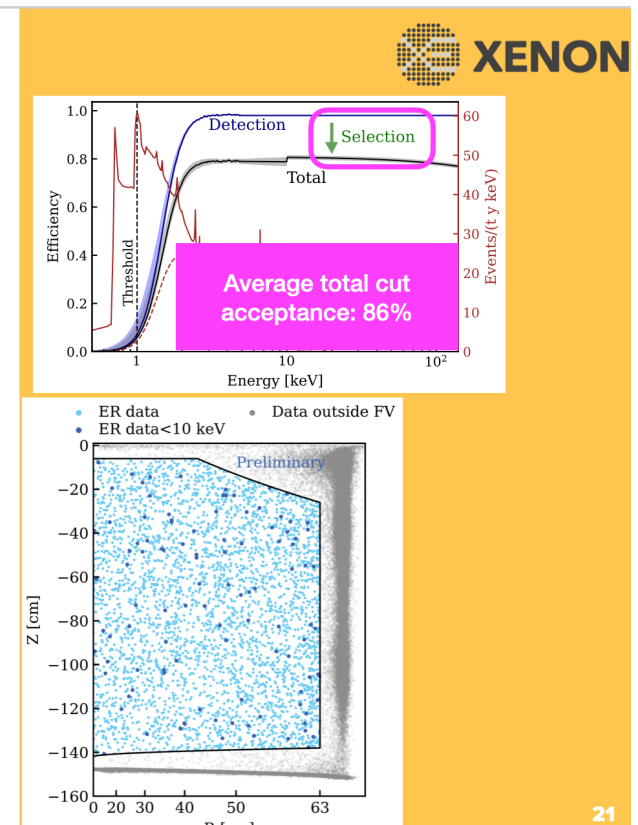
*“I am powerful. And I am only the most lowly gatekeeper. But from room to room stand gatekeepers, each more **powerful** than the other. I can’t endure even one glimpse of the third.”*

CUTS ARE OFTEN GOF TESTS!

- Many event selections may be considered goodness-of-fit tests—asking whether they are compatible with coming from a signal
- Others are more standard hypothesis tests, if the background model is specified
- But we often define some cuts first and only model what remains!

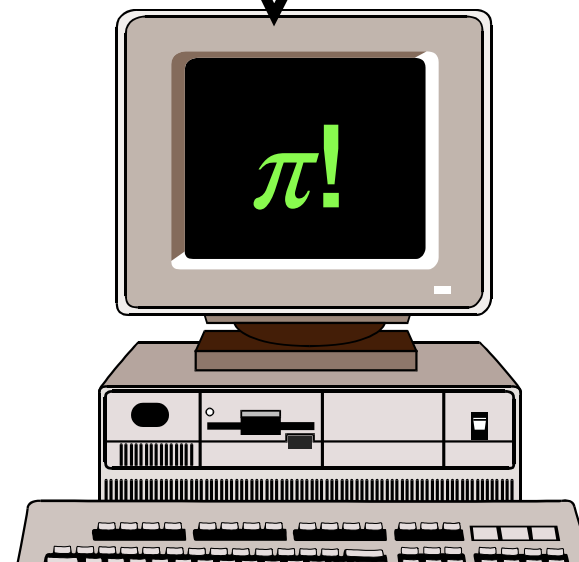
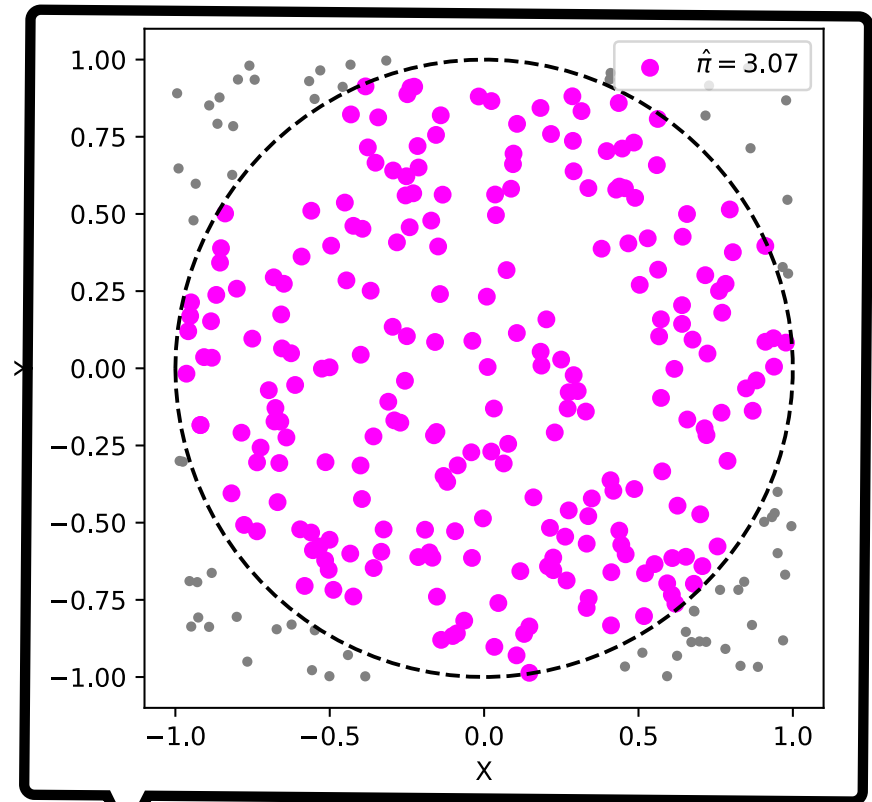
DATA QUALITY CUTS

- Events are required to pass a range of quality cuts:
 - The S1 and S2 peak should each have patterns, top/bottom ratios etc. consistent with real events
 - An S2 width consistent with the expected diffusion
 - An S2 over 500 PE
 - Not within < 300 ns of a neutron veto event
- Events must be within ER band
- Fiducial volume cut selects a mass of (4.37 ± 0.14) tonnes with low backgrounds

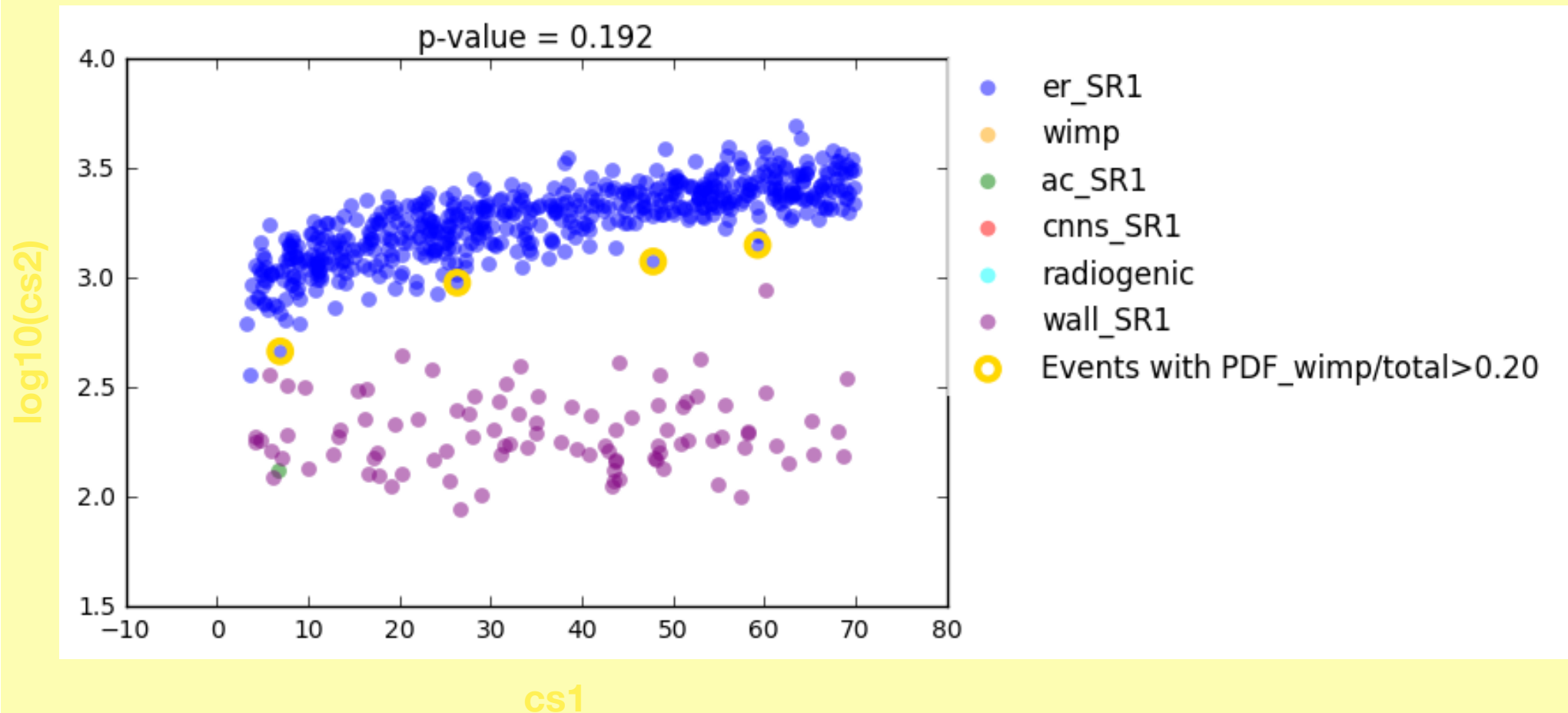
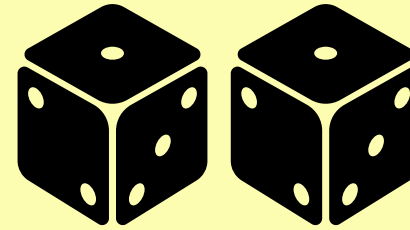


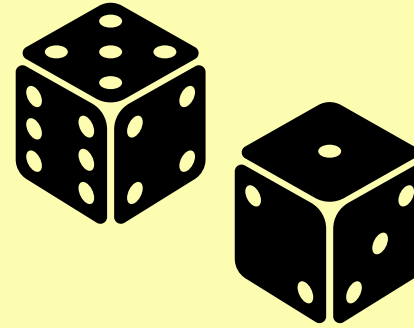
WHAT ARE TOY MONTE-CARLO METHODS?

- What is the area of a circle?
- Or, often equally importantly—what is the distribution of our estimate for π , or any other test statistic you can imagine?
- In this case you can figure out the distribution,
- But for many more complicated cases, you may either rely on approximations or simulated results

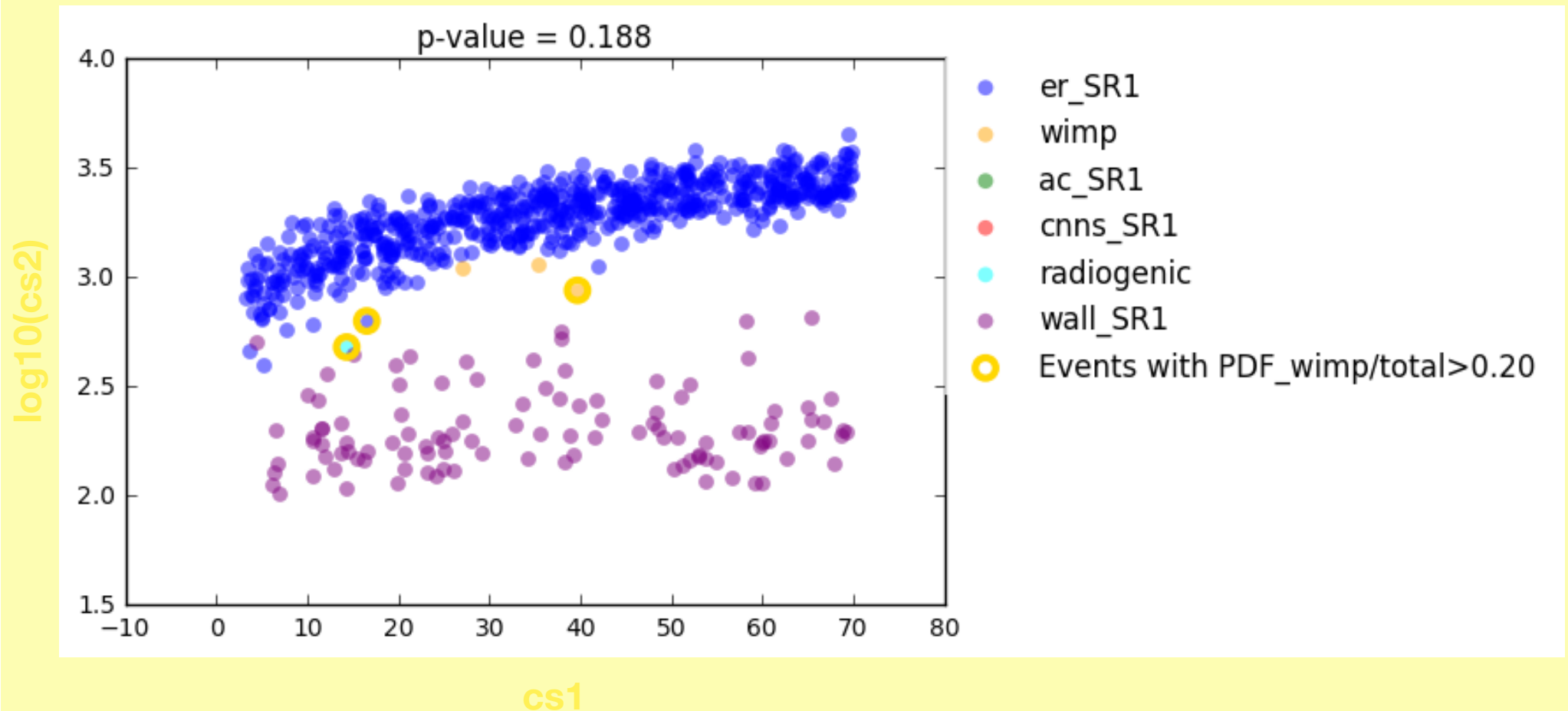


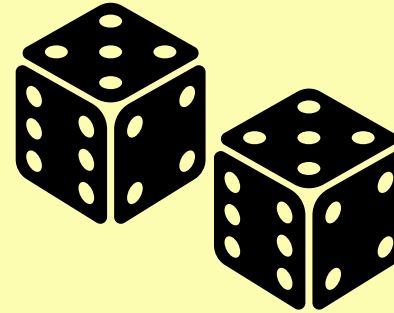
SEARCHING FOR DM IS A MATTER OF LUCK



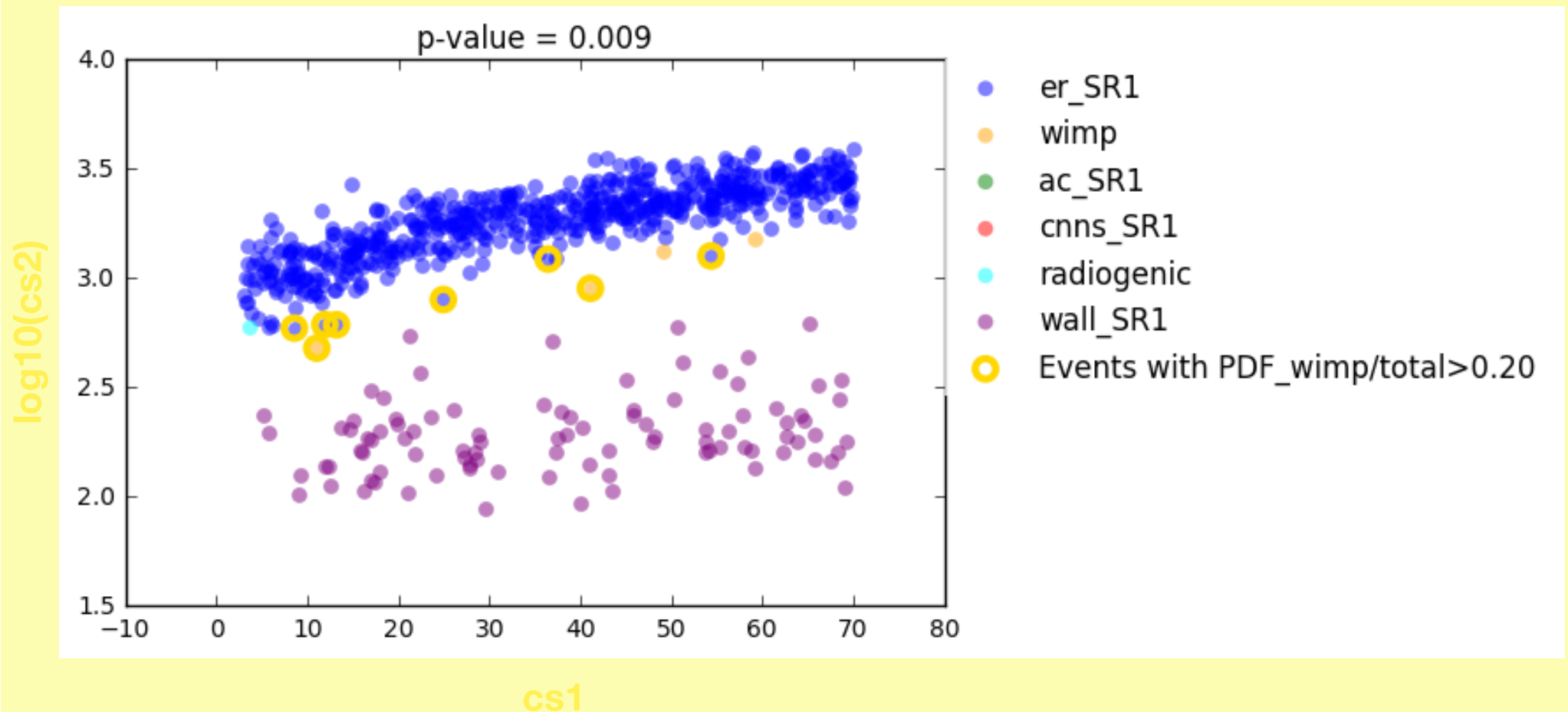


SEARCHING FOR DM IS A MATTER OF LUCK



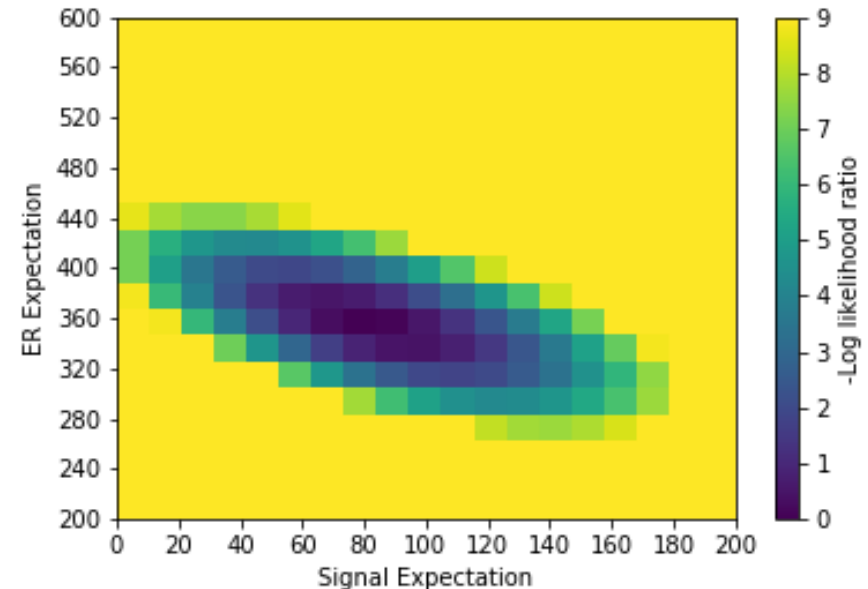


SEARCHING FOR DM IS A MATTER OF LUCK



PROFILING LIKELIHOODS & NUISANCE PARAMETERS

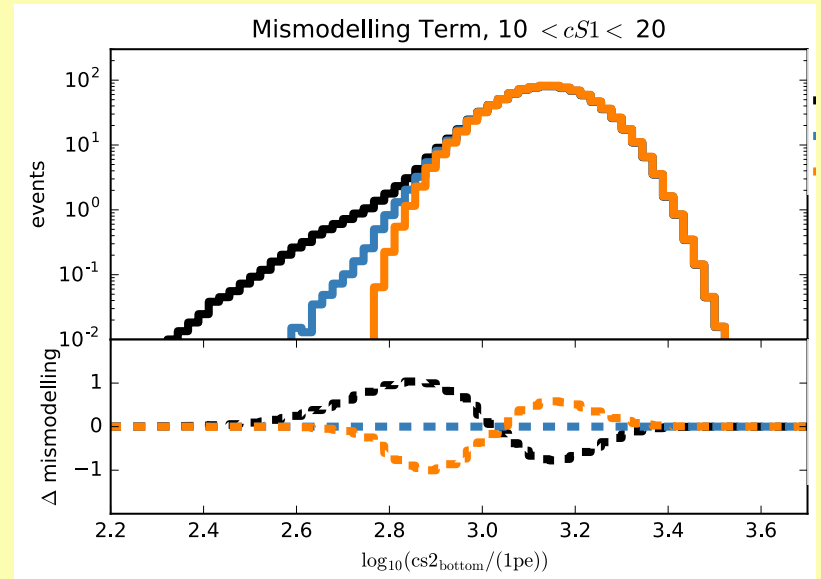
- We seldom have completely specified hypotheses
- Our background and signal models have uncertainties, parameterised by nuisance parameters (θ)— you'll see some examples in the next slides.
- The global best fit we denote with $\hat{s}, \hat{\theta}$
- However, we also want to test other s — for example $s=0$ for discovery significance or a range of s for confidence intervals.
- In these cases, we set the other nuisance parameters to their conditional best-fit $\hat{\theta}$.



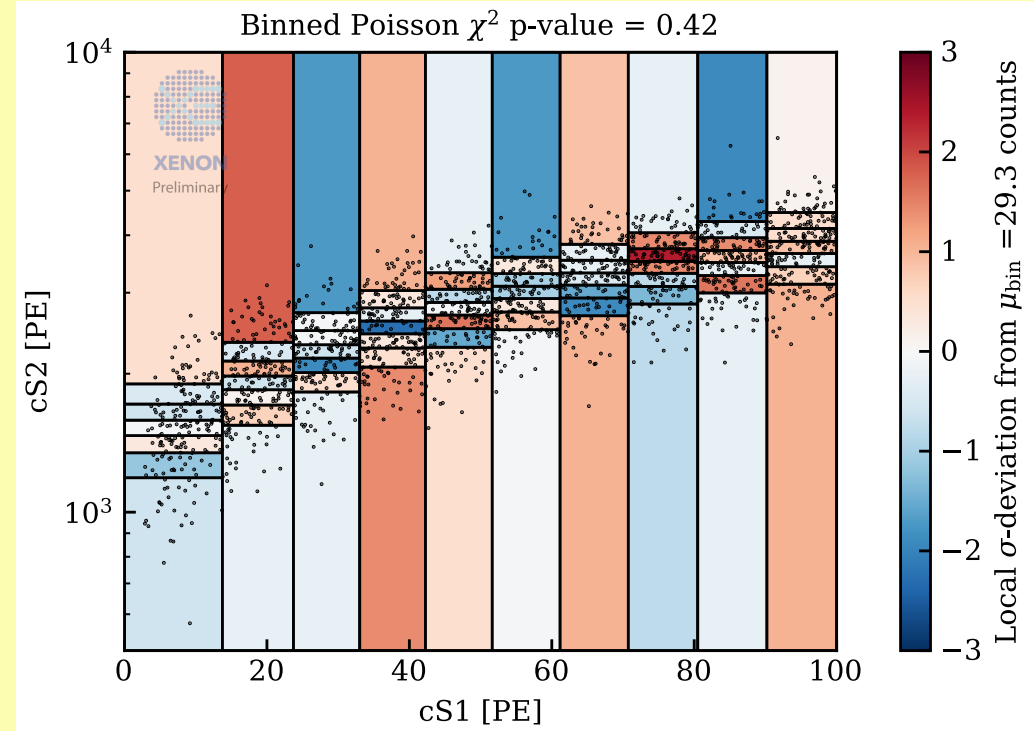
$$\delta \log \mathcal{L}(s, \hat{\theta}) / \delta \theta_j = 0;$$

THE LIKELIHOOD RELIES ON THE MODEL

- The validity of the inference relies on the underlying model
- The signal model may be quite forgiving— if an excess is 10-20 events, far tails are less significant
- Experiments typically include uncertainties on background rates, but not always on the distribution used.
- XENON1T added a “signal-like” background shape to its ER background model to lower the chance of overconstraining the model.
- For XENONnT, this was replaced by a more careful selection of nuisance parameter directions, and a stronger focus on pre-defined goodness-of-fit tests chosen for their power to discover mismodelling

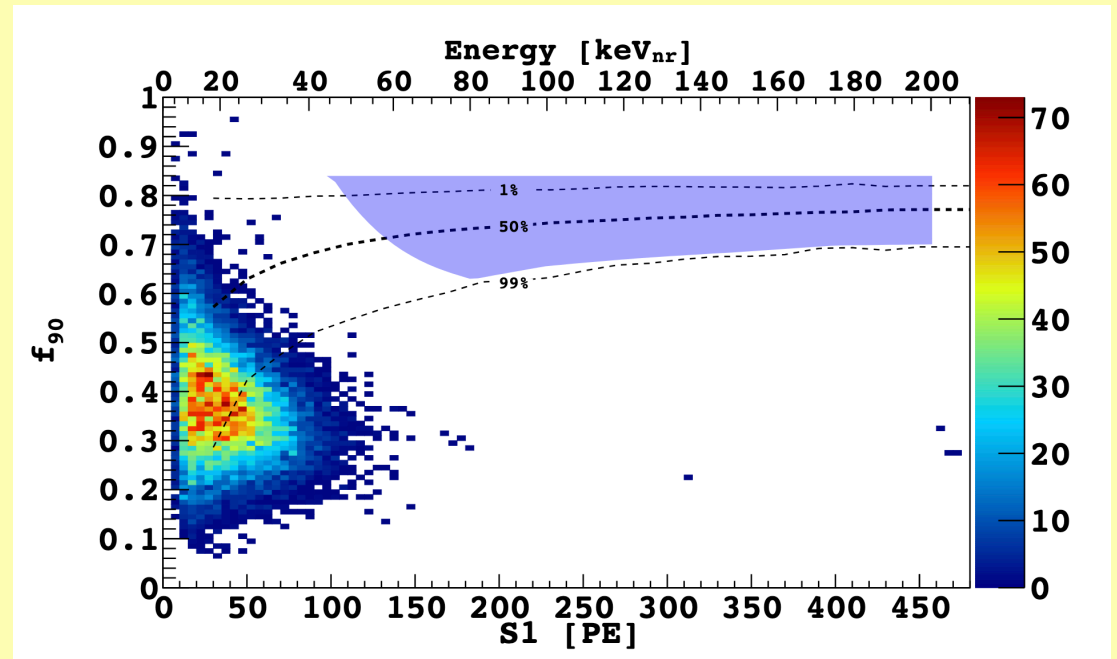


N. Priel et al. A model independent safeguard against background mismodeling for statistical inference. 2017(05):013–013, may 2017. doi: 10.1088/1475-7516/2017/05/013.



COUNTING EXPERIMENTS

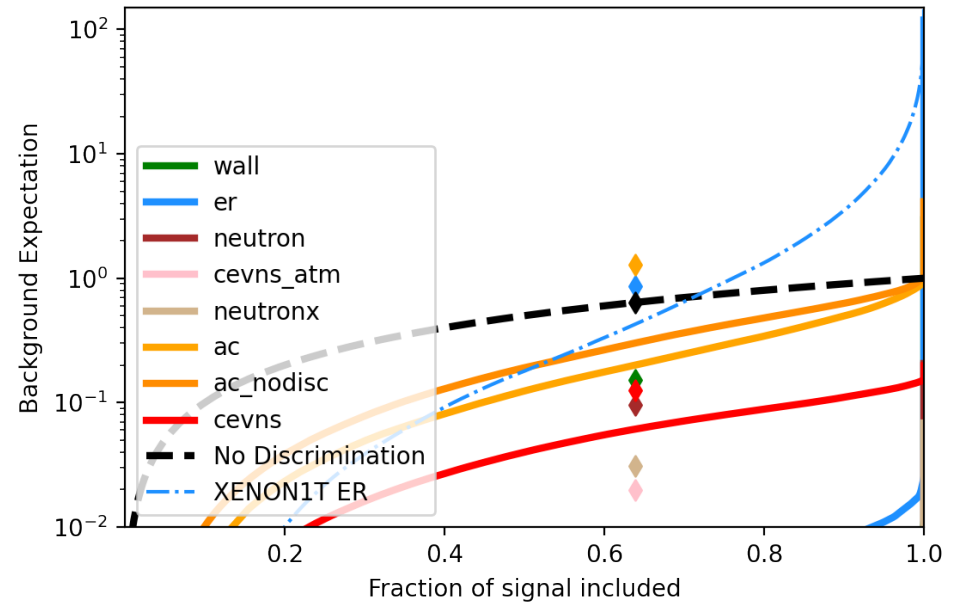
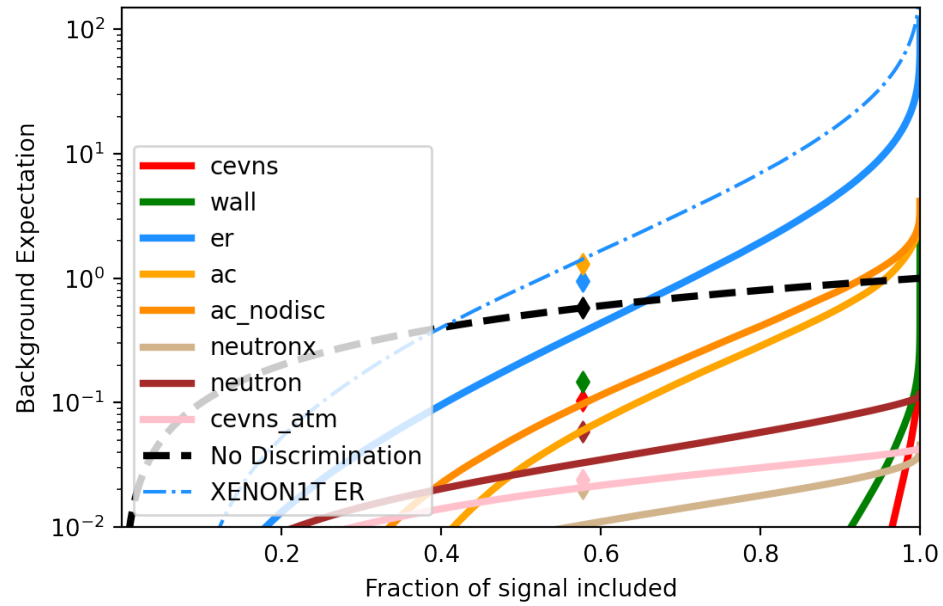
- “just” counting events— but the estimate of the background rate and acceptance can be as complicated as anything
- If there is no signal/background overlap *or* complete overlap, this may be the optimal sensitivity
- Otherwise, it might still be a worthwhile compromise if you’re worried about whether you can model your background correctly



DarkSide-50 532-day <https://arxiv.org/pdf/1802.07198>

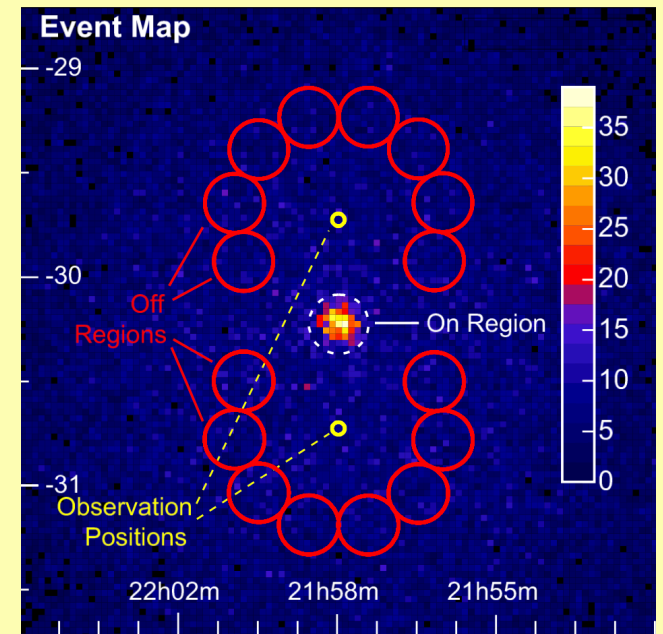
$$\mathcal{L}_{\text{sci}}(s, \vec{\theta}_s, \vec{\theta}_b) = \text{Poisson}(N_{\text{sci}} | \mu_b(\vec{\theta}_b) + \mu_s(s, \vec{\theta}_s, \vec{\theta}_b))$$

HOWEVER, SHAPES OFTEN MATTER



ON-OFF LIKELIHOODS

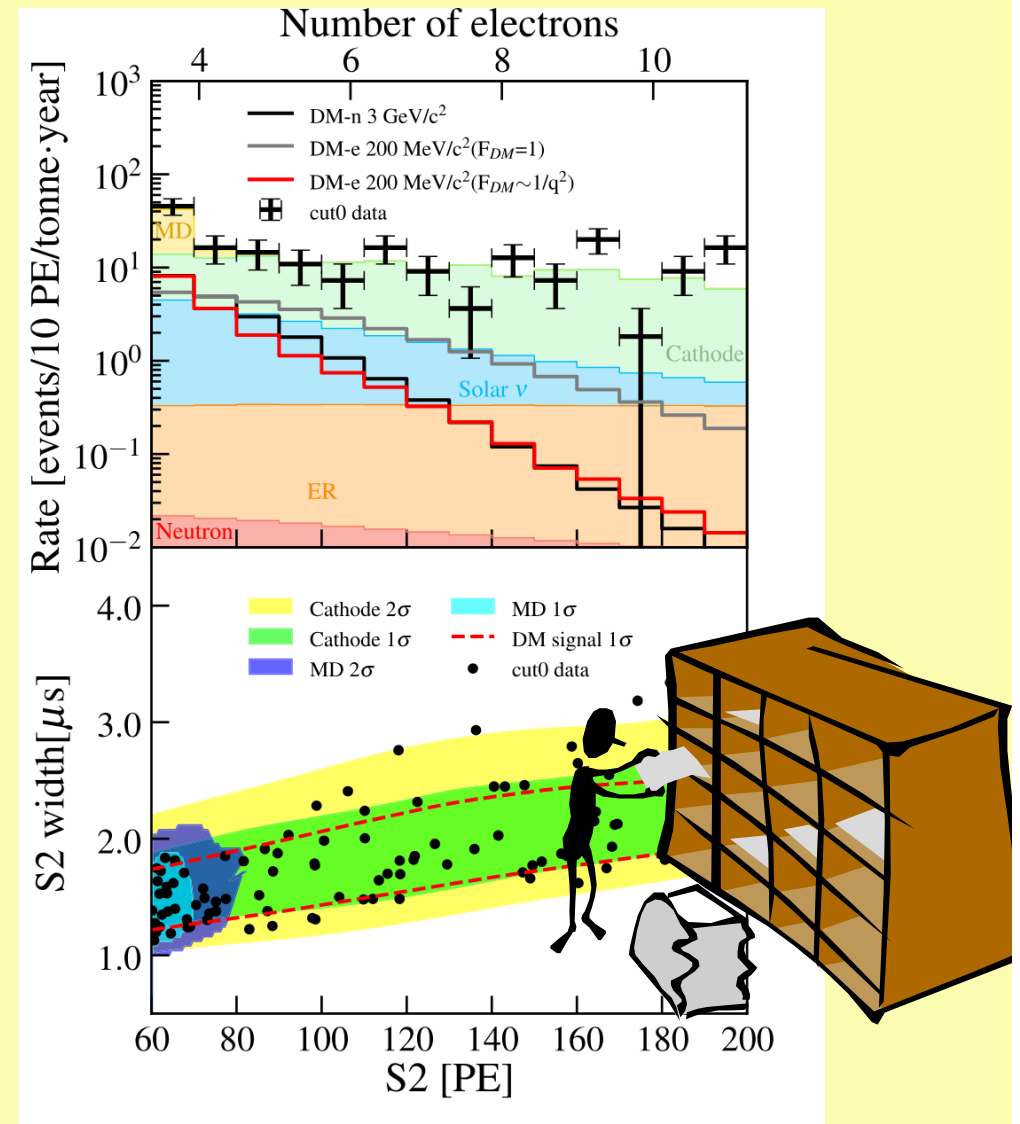
- WIMP searches rarely get to turn off their signal completely
- Directional dark matter searches and some axion searches, on the other hand can take representative data in a no/low signal and high signal state
- Also common in indirect detection



$$\mathcal{L}_{\text{sci}}(s, \vec{\theta}_s, \vec{\theta}_b) =$$
$$\text{Poisson}(N_{\text{sci}} | \mu_b(\vec{\theta}_b) + \mu_s(s, \vec{\theta}_s, \vec{\theta}_b)) \times$$
$$\text{Poisson}(N_{\text{cal}} | \alpha \times \mu_b(\vec{\theta}_b))$$

BINNED LIKELIHOOD

- With more than ~ 5 events in each bin, you can use computationally efficient methods to compute test statistic distributions
- Eases visualisation and goodness-of-fit
- And simpler to share results
- Minimal sensitivity loss if the bin width is small compared to the detector resolution

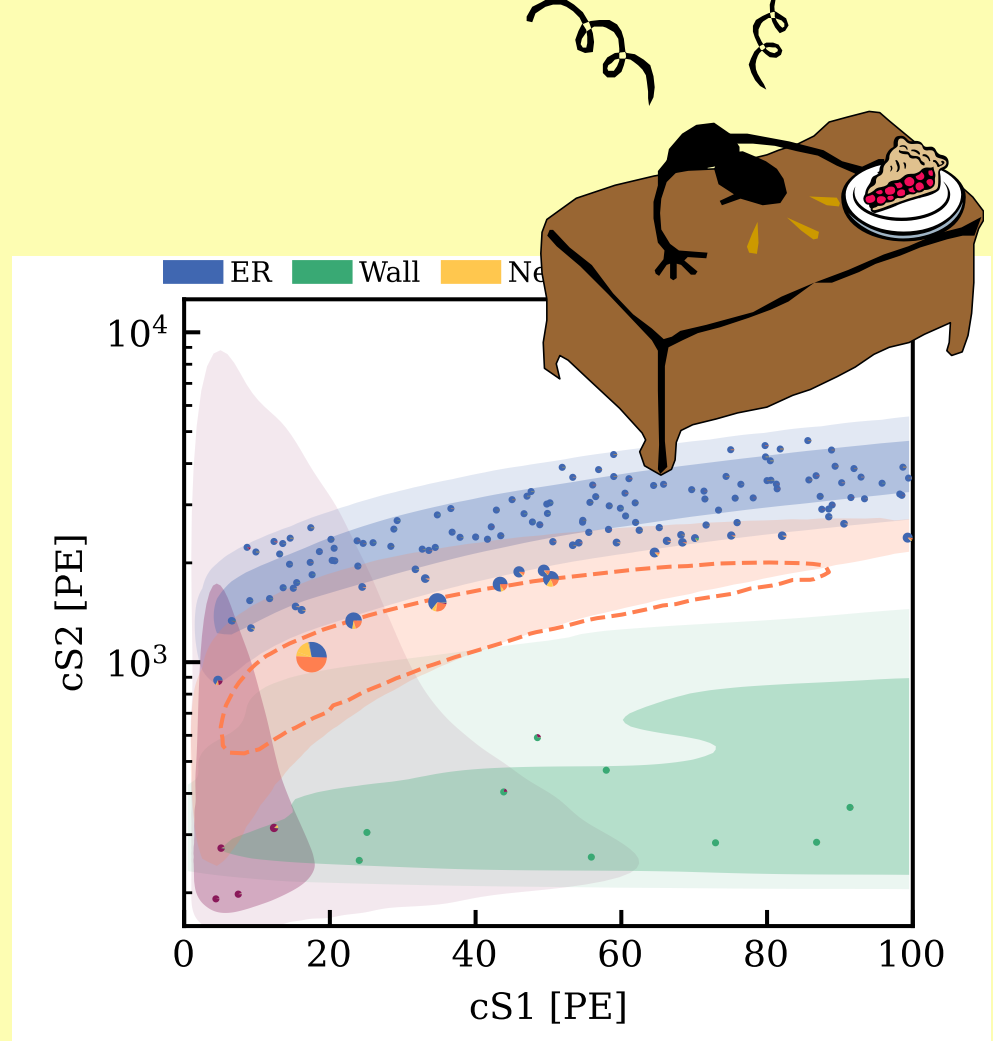


PandaX ionisation-only search, <https://arxiv.org/abs/2212.10067>

$$\mathcal{L}_{\text{sci}}(s, \vec{\theta}_s, \vec{\theta}_b) = \prod_{i=1}^{N_s} \left[\text{Poisson}(N_i | \mu_{b,i}(\vec{\theta}_b) + \mu_{s,i}(s, \vec{\theta}_s, \vec{\theta}_b)) \right]$$

UNBINNED (EXTENDED) LIKELIHOOD

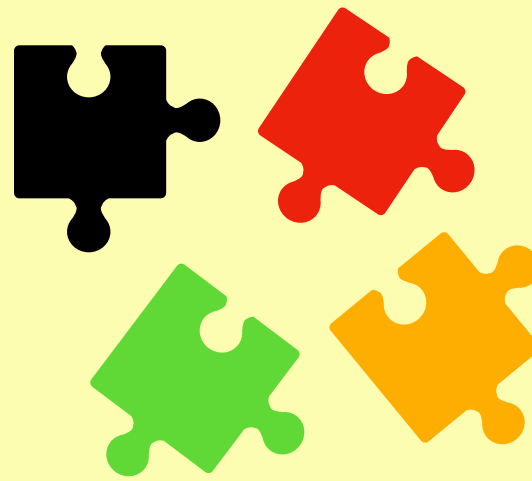
- If the events are too few to fill bins, the unbinned likelihood promises the best performance
- Might still have to rely on binned methods for goodness-of-fit
- if you rely on Monte Carlo methods to generate distributions, that can require a lot of statistics and be harder to validate



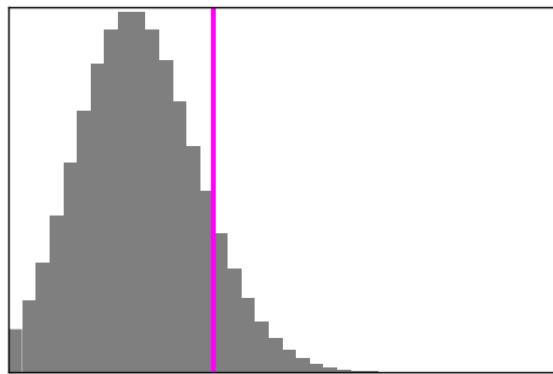
XENONnT first WIMP search

$$\mathcal{L}_{\text{sci}}(s, \vec{\theta}_s, \vec{\theta}_b) = \text{Poisson}(N_{\text{sci}} | \mu_b(\vec{\theta}_b) + \mu_s(s, \vec{\theta}_s, \vec{\theta}_b)) \times \prod_{i=1}^{N_s} \left[\frac{\mu_s}{\mu_s + \mu_b} f_s(\vec{x}_i | s, \vec{\theta}_s, \vec{\theta}_b) + \frac{\mu_b}{\mu_s + \mu_b} f_b(\vec{x}_i | \vec{\theta}_b) \right]$$

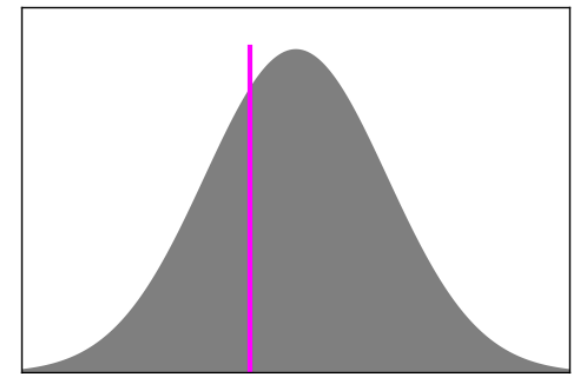
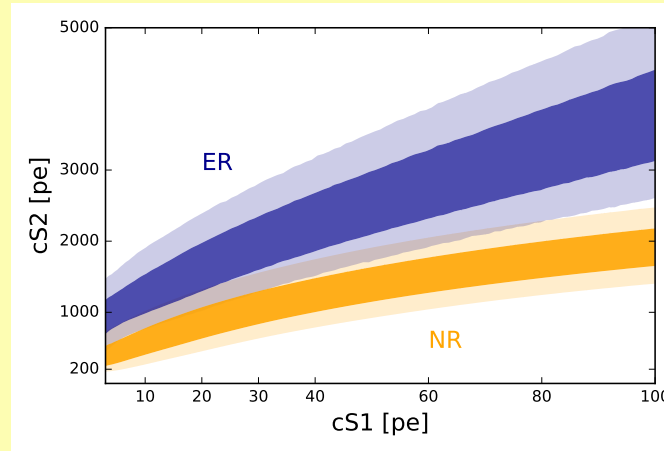
Likelihoods can be composed



$$\mathcal{L}(s, \vec{\theta}_s, \vec{\theta}_b)_{\text{Science run}} = \mathcal{L}_{\text{sci}}(s, \vec{\theta}_s, \vec{\theta}_b) \times \mathcal{L}_{\text{cal}}(\vec{\theta}_b) \times \mathcal{L}_{\text{anc}}(\vec{\theta}_b)$$



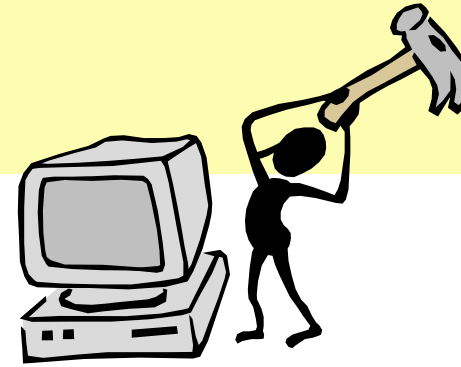
Number of Events



Nuisance Parameter

$$\mathcal{L}(s, \vec{\theta}_s, \vec{\theta}_b)_{\text{tot}} = \mathcal{L}(s, \vec{\theta}_s, \vec{\theta}_b)_{\text{tot}} \times \mathcal{L}(s, \vec{\theta}_s, \vec{\theta}_b)_{\text{tot}} \times \mathcal{L}_{\text{shared}}(\theta)$$

ASYMPTOTIC DISTRIBUTIONS



- The log-likelihood for a number of gaussian-distributed numbers has the same form as the χ^2 -formula (Wilks' theorem)
- It turns out that if a set of conditions that are quite oftenTM fulfilled, the distribution of the likelihood ratio converges to a χ^2 -distribution with some number of free parameters
- This can massively simplify your computations, and so it is worth to look through in detail

Necessary conditions for Wilks' theorem

ASYMPTOTIC: Sufficient data is observed.

INTERIOR: Only values of μ and θ which are far from the boundaries of their parameter space are admitted.

IDENTIFIABLE: Different values of the parameters specify distinct models.

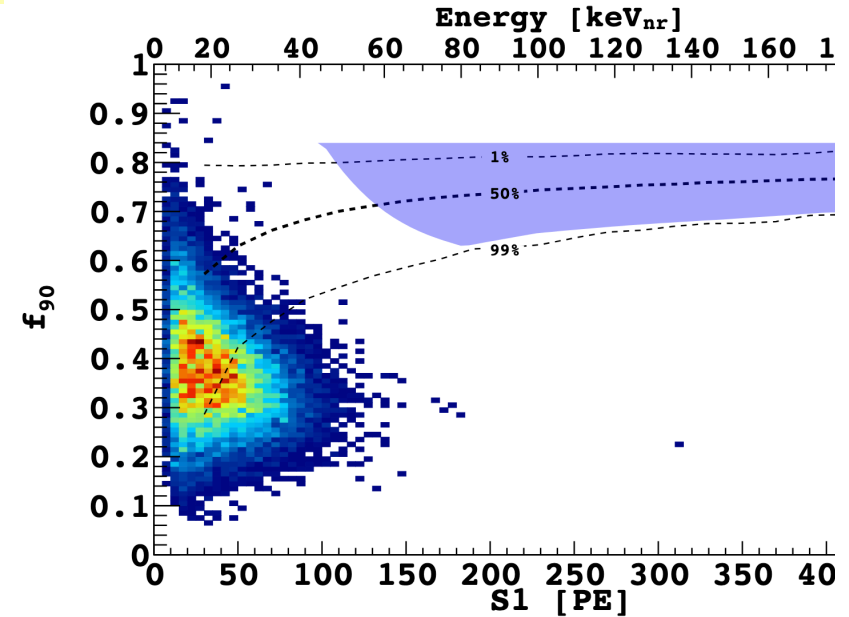
NESTED: H_0 is a limiting case of H_1 , e.g. with some parameter fixed to a sub-range of the entire parameter space.

CORRECT: The true model is specified either under H_0 or under H_1 .

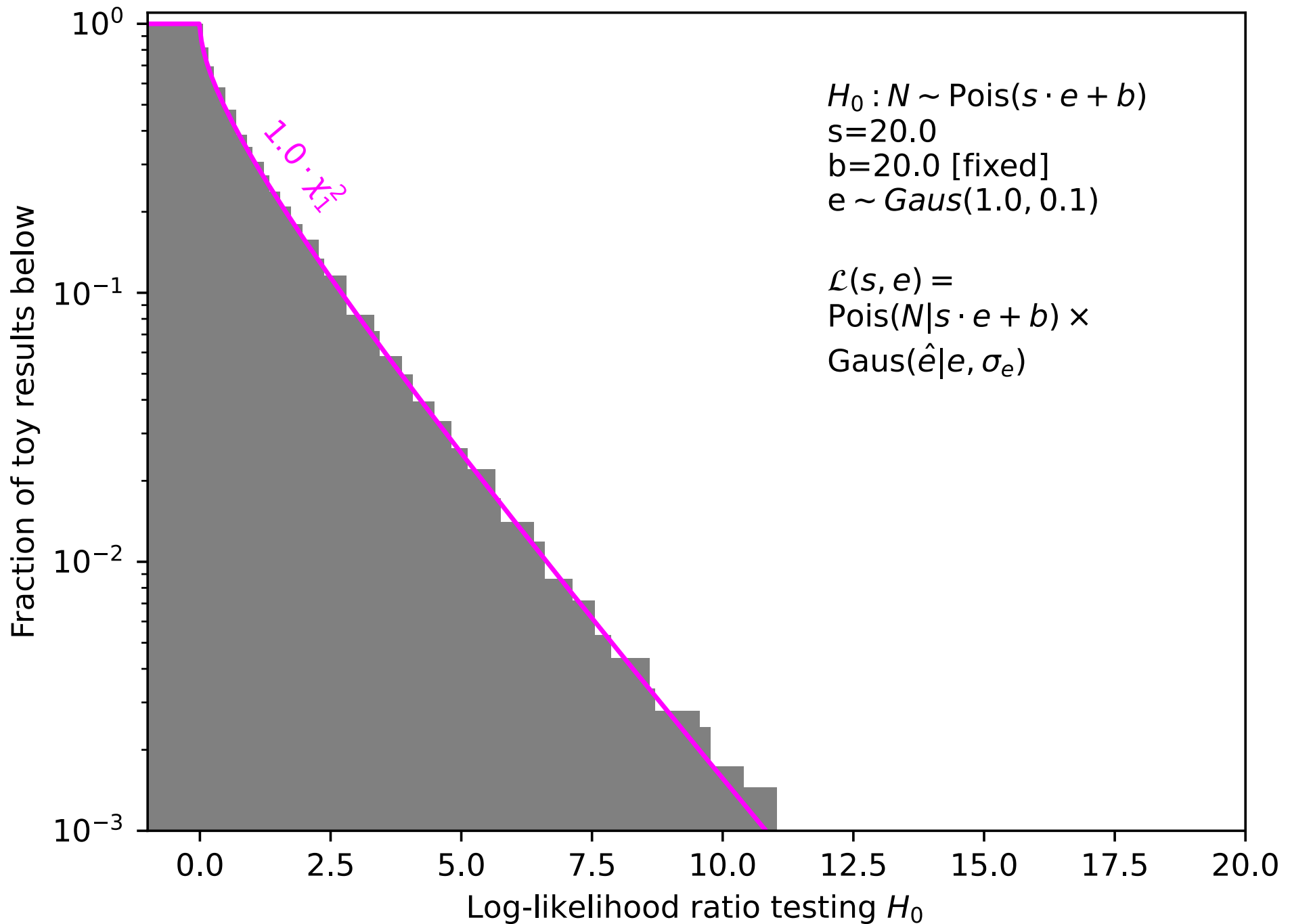
$$q(s) = -2 \cdot \log\left(\frac{\mathcal{L}(s, \hat{\theta})}{\mathcal{L}(\hat{s}, \hat{\theta})}\right)$$

ASYMPTOTIC DISTRIBUTIONS

- As our example: the profile log-likelihood ratio test for a counting experiment with a known background but uncertain efficiency
- Parameters:
 - Signal s
 - efficiency e
- Fixed, known parameters:
 - Background expectation b
 - efficiency uncertainty σ_e
- Data:
 - Number of events N
 - efficiency estimate e_{meas}



$$(L)(s, e) = \text{Pois}(N | s \cdot e + b) \times \text{Gaus}(e_{\text{meas}} | e, \sigma_e)$$



ASYMPTOTIC DISTRIBUTIONS

- Wilks' theorem holds in the asymptotic case of infinite data, but convergence can often be quick:
 - Poisson counting with more than ca. 10 events
 - Gaussian measurements
- However, if you have an unbinned likelihood, the important consideration is *signal-like* background events— for example seen with IXe TPC searches

Necessary conditions for Wilks' theorem

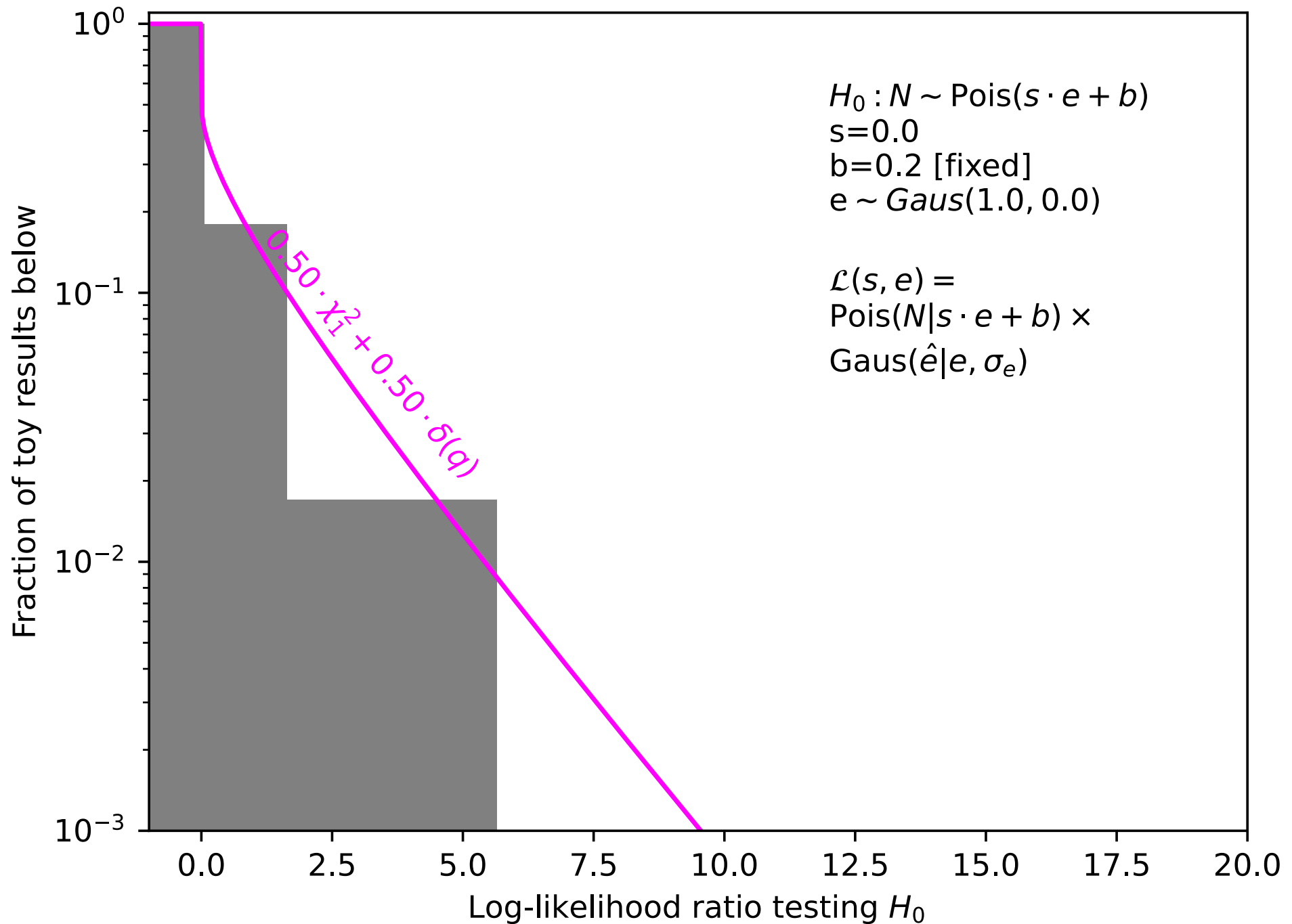
ASYMPTOTIC: Sufficient data is observed.

INTERIOR: Only values of μ and θ which are far from the boundaries of their parameter space are admitted.

IDENTIFIABLE: Different values of the parameters specify distinct models.

NESTED: H_0 is a limiting case of H_1 , e.g. with some parameter fixed to a sub-range of the entire parameter space.

CORRECT: The true model is specified either under H_0 or under H_1 .



ASYMPTOTIC DISTRIBUTIONS

- As a mental shortcut— if under your null or signal hypothesis, parameters sometimes or often goes to a physical boundary, it will not behave asymptotically
- This is very often the case e.g. if you're looking for a signal with expectation value ≥ 0
- If you are testing the hypothesis that the model that has the parameter *at the boundary*— for example that the signal is 0, you may be able to use *Chernoff's theorem* if all other conditions are met

Necessary conditions for Wilks' theorem

ASYMPTOTIC: Sufficient data is observed.

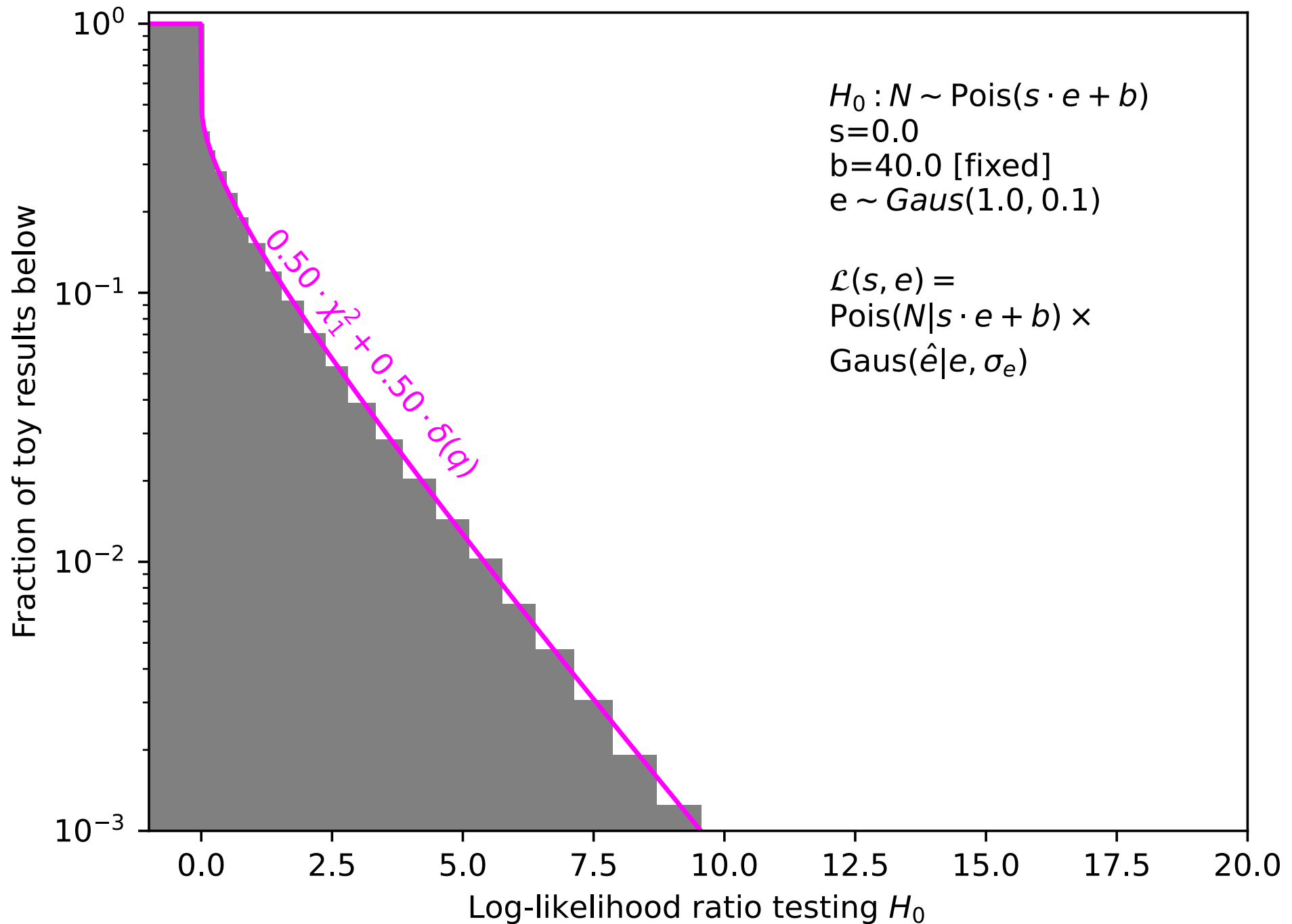
INTERIOR: Only values of μ and θ which are far from the boundaries of their parameter space are admitted.

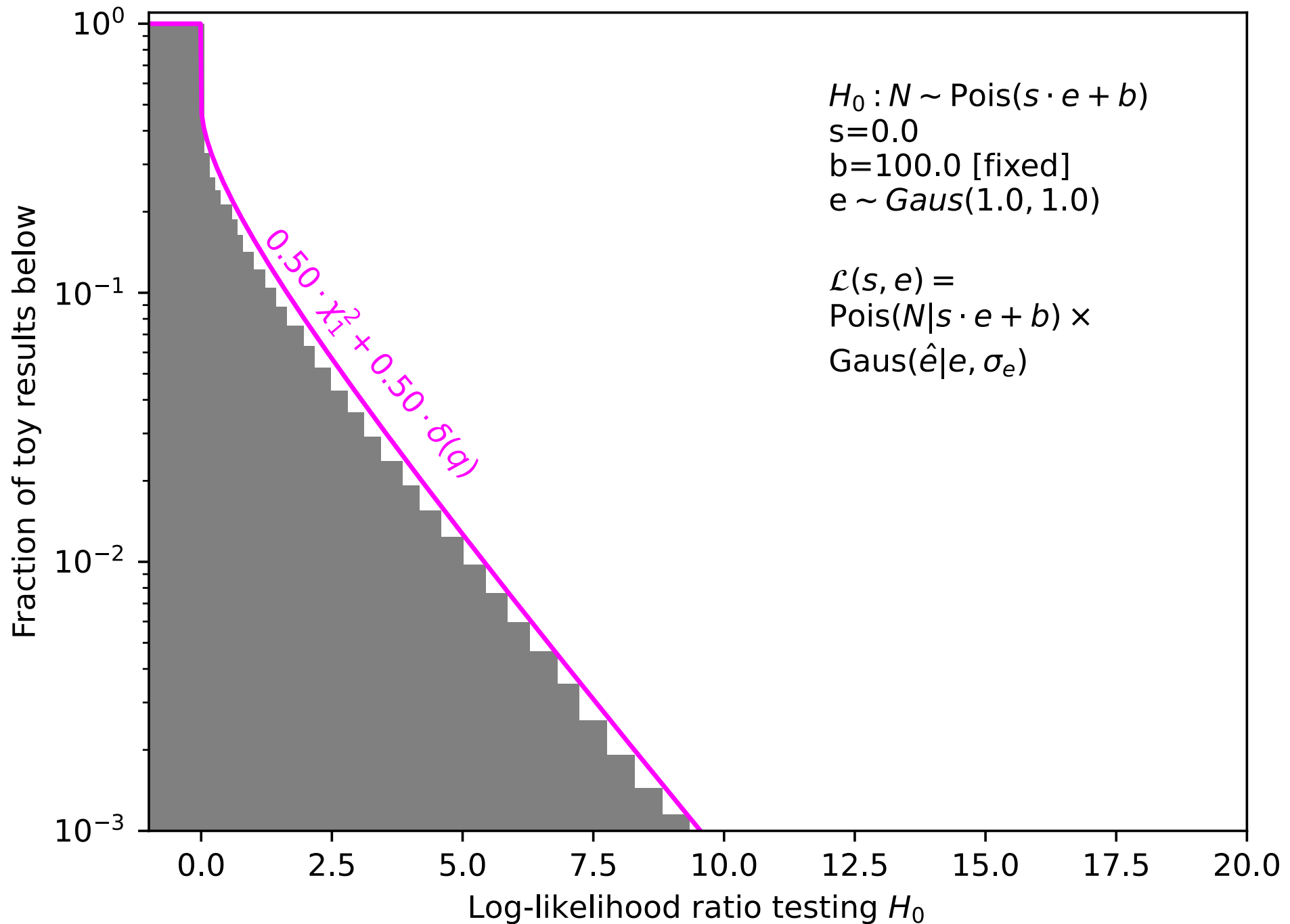
IDENTIFIABLE: Different values of the parameters specify distinct models.

NESTED: H_0 is a limiting case of H_1 , e.g. with some parameter fixed to a sub-range of the entire parameter space.

CORRECT: The true model is specified either under H_0 or under H_1 .

$$f(q) \stackrel{\text{Chernoff}}{\approx} \frac{1}{2} \chi_{DOF=1}^2 + \frac{1}{2} \delta(\hat{\mu})$$





ASYMPTOTIC DISTRIBUTIONS

- If the model is degenerate for some parameter, the asymptotic approximation will not hold
- This is quite common in physics! When the signal strength is 0, the model does not depend on any other signal parameter
- This is another way of looking at the look-elsewhere effect, which we'll look at later

Necessary conditions for Wilks' theorem

ASYMPTOTIC: Sufficient data is observed.

INTERIOR: Only values of μ and θ which are far from the boundaries of their parameter space are admitted.

IDENTIFIABLE: Different values of the parameters specify distinct models.

NESTED: H_0 is a limiting case of H_1 , e.g. with some parameter fixed to a sub-range of the entire parameter space.

CORRECT: The true model is specified either under H_0 or under H_1 .

ASYMPTOTIC DISTRIBUTIONS

- If the model tested is not a limit of the general hypothesis
 - Such as when testing between two disparate models
 - Or if your theory features a non-zero fixed signal you wish to test against the no-signal hypothesis
- You can always linearly add the two hypotheses' models together with a new parameter, but then you introduce Non-identifiability at the boundary!

Necessary conditions for Wilks' theorem

ASYMPTOTIC: Sufficient data is observed.

INTERIOR: Only values of μ and θ which are far from the boundaries of their parameter space are admitted.

IDENTIFIABLE: Different values of the parameters specify distinct models.

NESTED: H_0 is a limiting case of H_1 , e.g. with some parameter fixed to a sub-range of the entire parameter space.

CORRECT: The true model is specified either under H_0 or under H_1 .

ASYMPTOTIC DISTRIBUTIONS

- All our inference results are reliant on the true model being somewhere in our model space!
- However, we should be cognisant that this is never guaranteed
- If you have a mismodelling you are concerned about, you should test how much it can affect your results— you might well find that your method is robust to it, or you can add model uncertainties to represent this
- Another way to increase robustness is to make your model simpler— a counting experiment makes fewer assumptions on the energy spectrum than if you include the energy information

Necessary conditions for Wilks' theorem

ASYMPTOTIC: Sufficient data is observed.

INTERIOR: Only values of μ and θ which are far from the boundaries of their parameter space are admitted.

IDENTIFIABLE: Different values of the parameters specify distinct models.

NESTED: H_0 is a limiting case of H_1 , e.g. with some parameter fixed to a sub-range of the entire parameter space.

CORRECT: The true model is specified either under H_0 or under H_1 .

ASYMPTOTIC DISTRIBUTIONS

- Any gaussian-distributed measurements
 - Including histograms with high bin counts
- unbinned likelihoods with significant signal-like backgrounds
- Most common extra consideration is taking care of the parameter boundaries
- The below paper presents some cases:

$$q_{\text{discovery}} = \begin{cases} q(0) & \text{if } 0 \leq \hat{\mu} \\ 0 & \text{else} \end{cases}$$

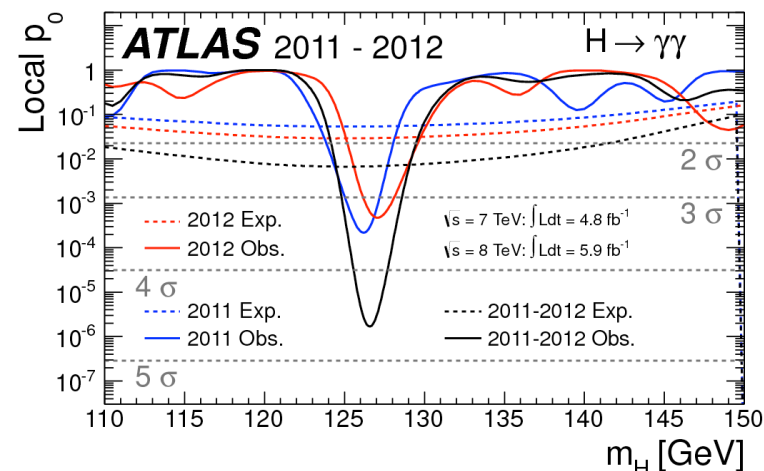
$$q_{\text{upper limit}}(\mu) = \begin{cases} q(\mu) & \text{if } \hat{\mu} \leq \mu \\ 0 & \text{else} \end{cases}$$

$$q(\mu)_{\text{unified}} = \begin{cases} -2 \cdot \frac{\mathcal{L}(\mu, \hat{\boldsymbol{\theta}})}{\mathcal{L}(\hat{\mu}, \hat{\boldsymbol{\theta}})} & \text{if } 0 \leq \hat{\mu} \\ -2 \cdot \frac{\mathcal{L}(\mu, \hat{\boldsymbol{\theta}})}{\mathcal{L}(0, \hat{\boldsymbol{\theta}}_{\mu=0})} & \text{else} \end{cases}$$

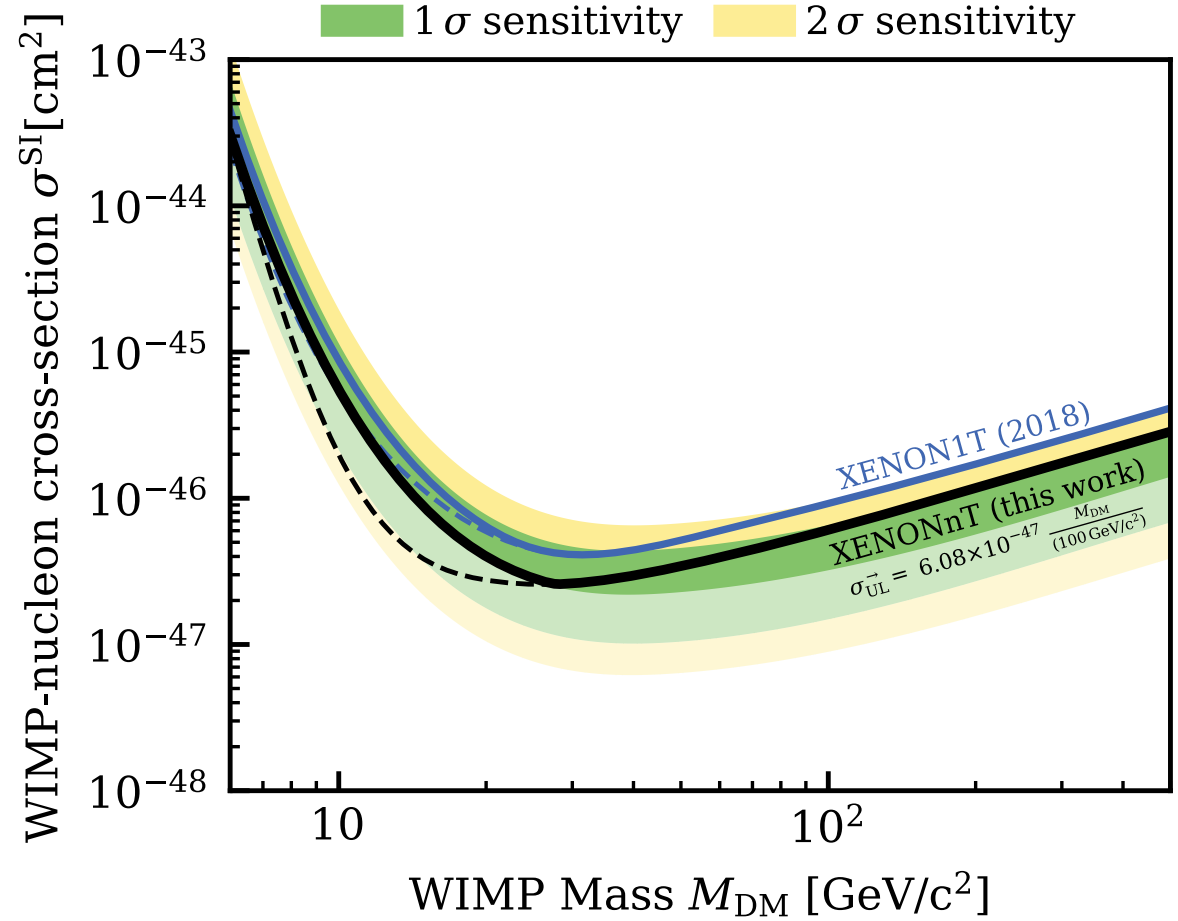
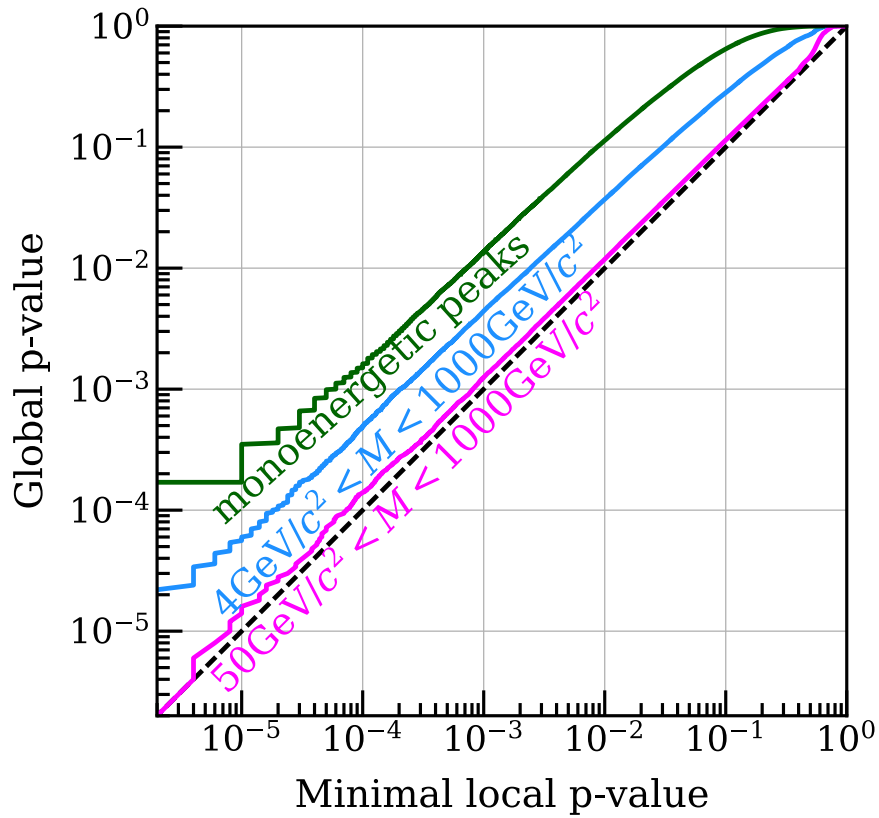
Note that these three can be seen as the same test statistic if you always restrict $\hat{\mu}, \mu$ to be positive!

THE LOOK ELSEWHERE EFFECT

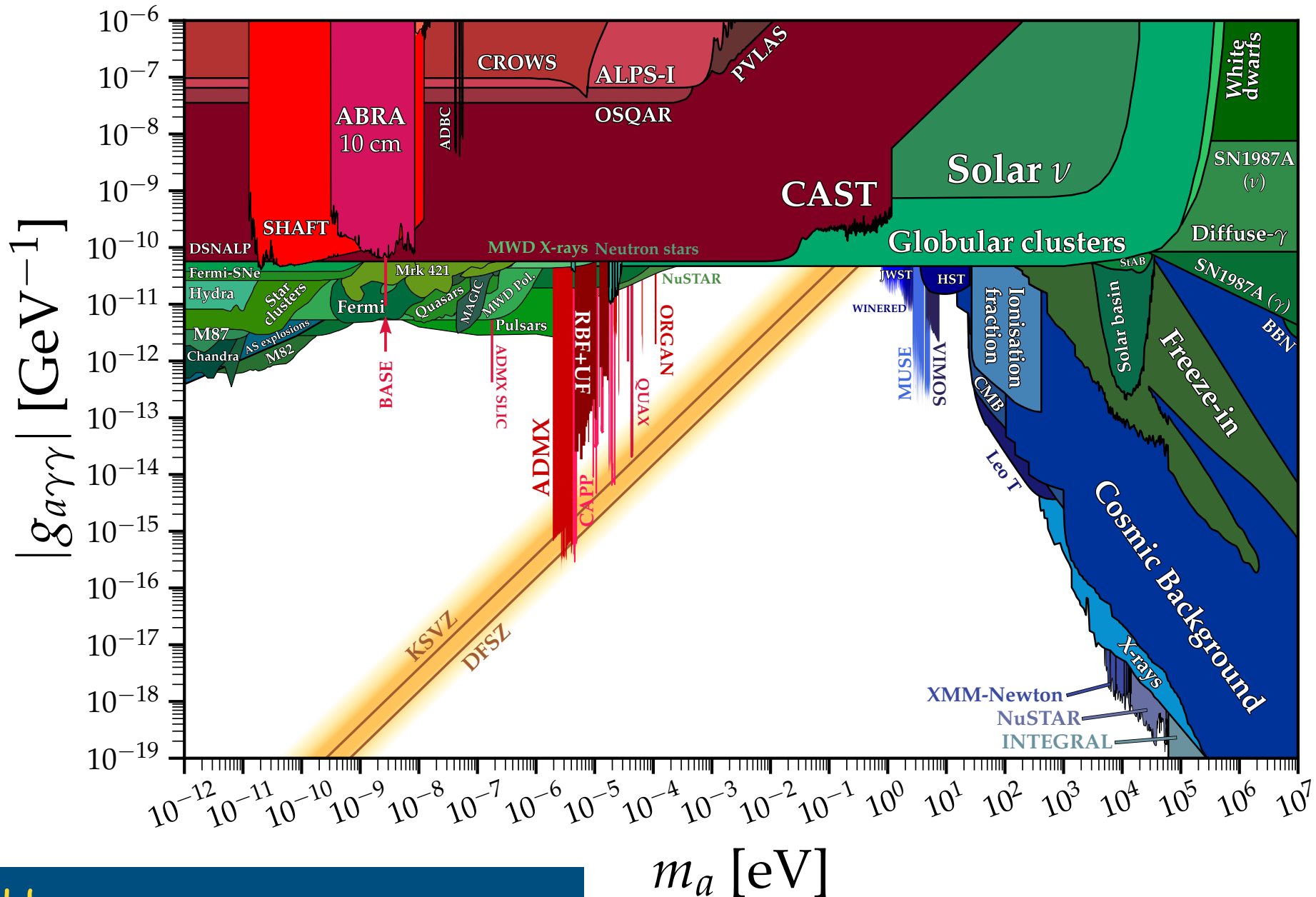
- Your probability to roll 6 on a dice increases the more dice you get to roll
- Similarly, if your experiment tests several signals, they will increase their chance to see unusual effects just by chance
- Separate between “local” significance— the probability that one single signal model tests fluctuates to some significance
- and “global” significance— the probability that *any* test fluctuates to that extent



THE LOOK ELSEWHERE EFFECT



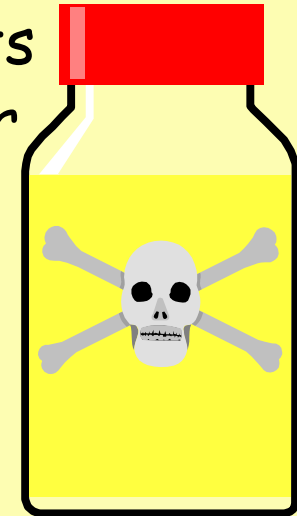
THE LOOK ELSEWHERE EFFECT



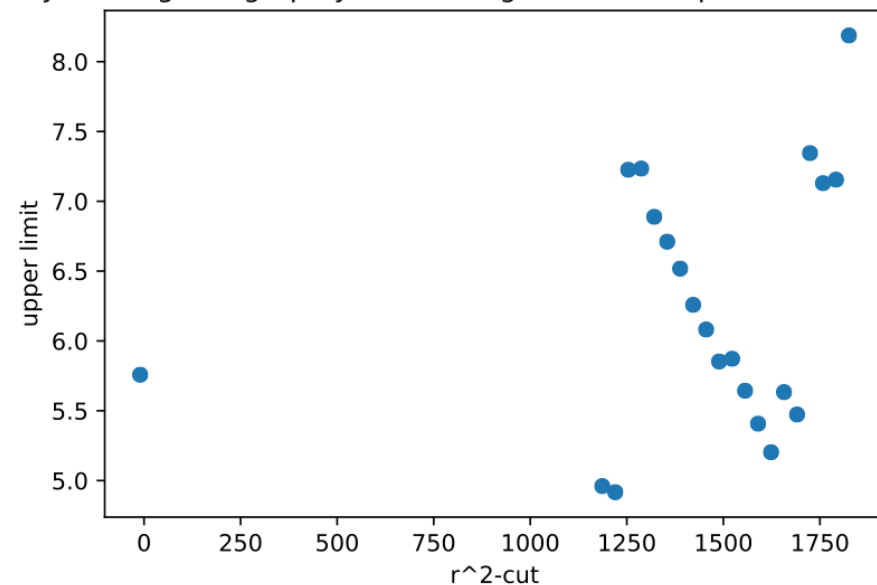
EXPERIMENTER BIAS IS A DANGER WITH FEW EVENTS

- With few events the effect can be drastic if you chance something in your analysis— the plot shows the 60% change in limit available to you between the best post-unblinding and the worst post-unblinding radial cut.
- This is a necessary consequence of making your analysis sensitive to few events!
- Further, with only some hundreds of events, and many variables, every event may well be an outlier in some space

Homeopathic poison
— the fewer events
the greater danger

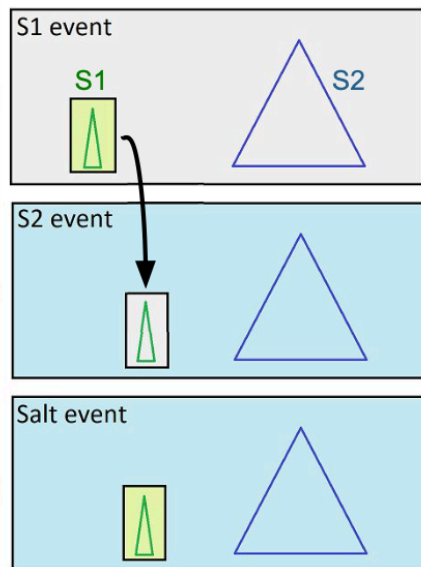


by viewing this graph you are obligated not to optimise based on it

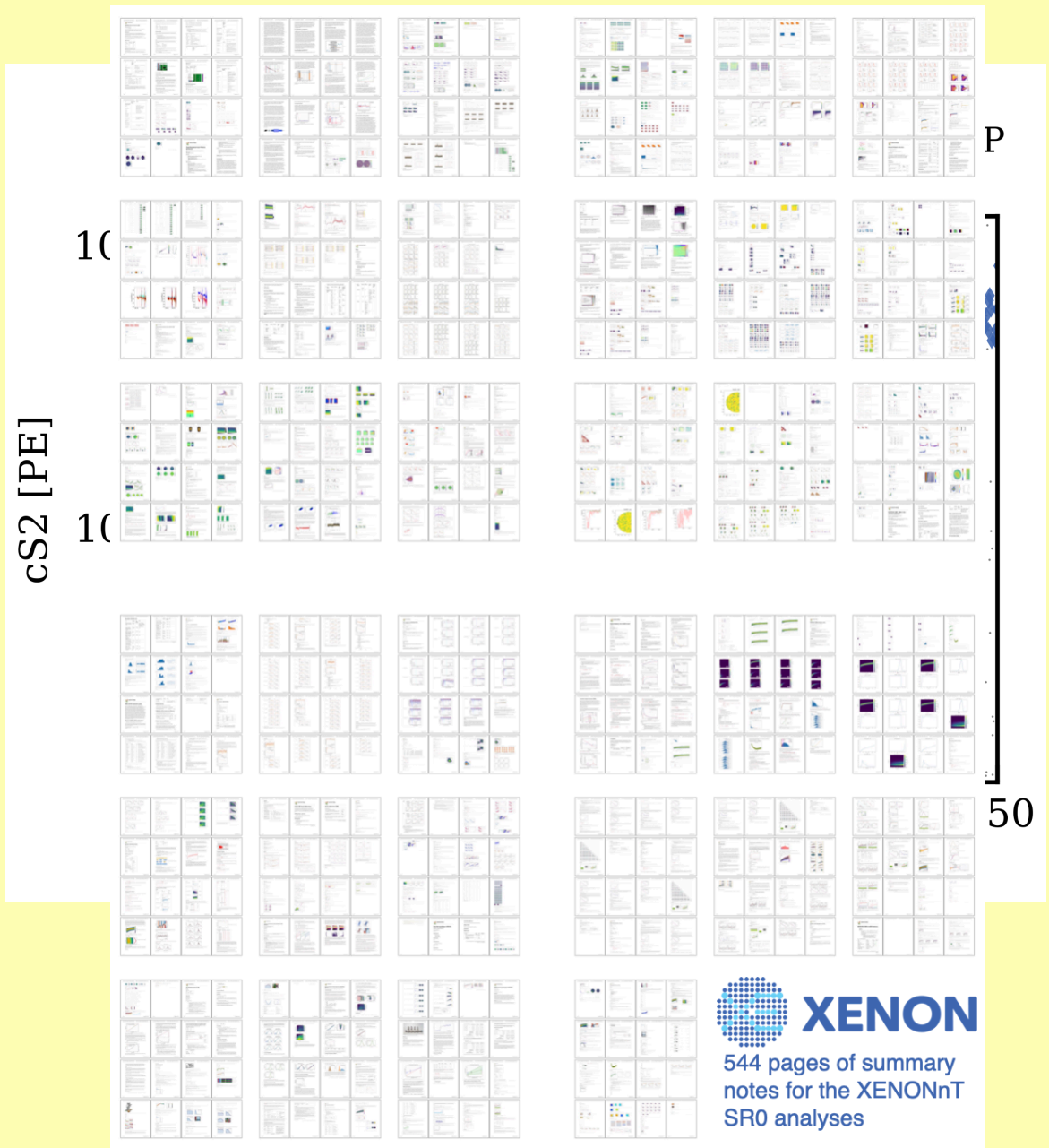


EXPERIMENTER BIAS IS A DANGER WITH FEW EVENTS

- The most common experimenter bias mitigation method is “blinding” — not showing the signal-like region of parameter space until the analysis has been frozen
- LUX developed a “salting” procedure where synthetic signals were made by stitching together genuine S1 and S2 signals into full events and placing them in the

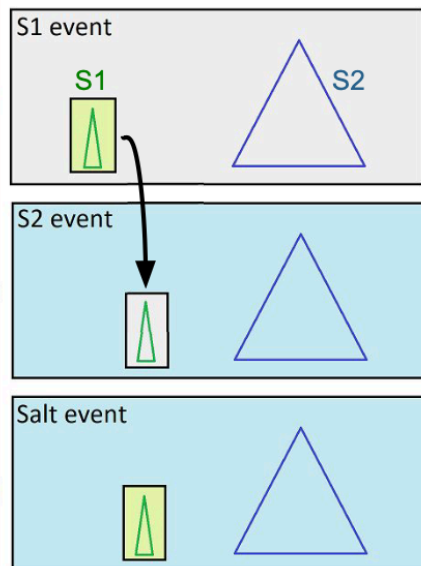


Tyler Anderson “Salting as a Bias Mitigation Technique in LZ”, presentation at LIDINE 2021

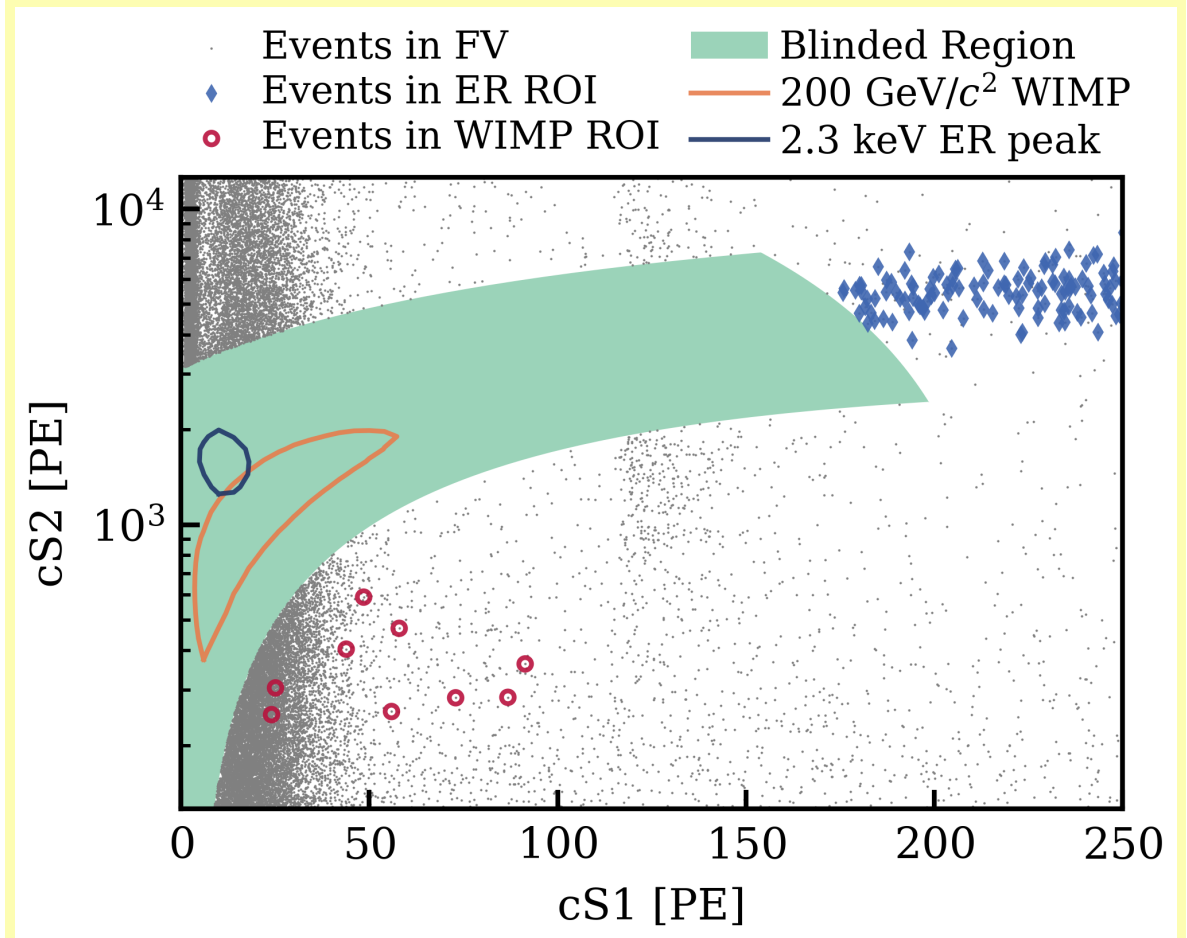


EXPERIMENTER BIAS IS A DANGER WITH FEW EVENTS

- The most common experimenter bias mitigation method is “blinding” — not showing the signal-like region of parameter space until the analysis has been frozen
- LUX developed a “salting” procedure where synthetic signals were made by stitching together genuine S1 and S2 signals into full events and placing them in the

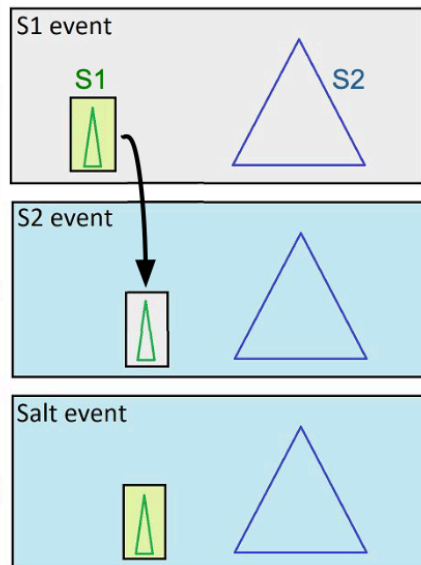


Tyler Anderson “Salting as a Bias Mitigation Technique in LZ”, presentation at LIDINE 2021

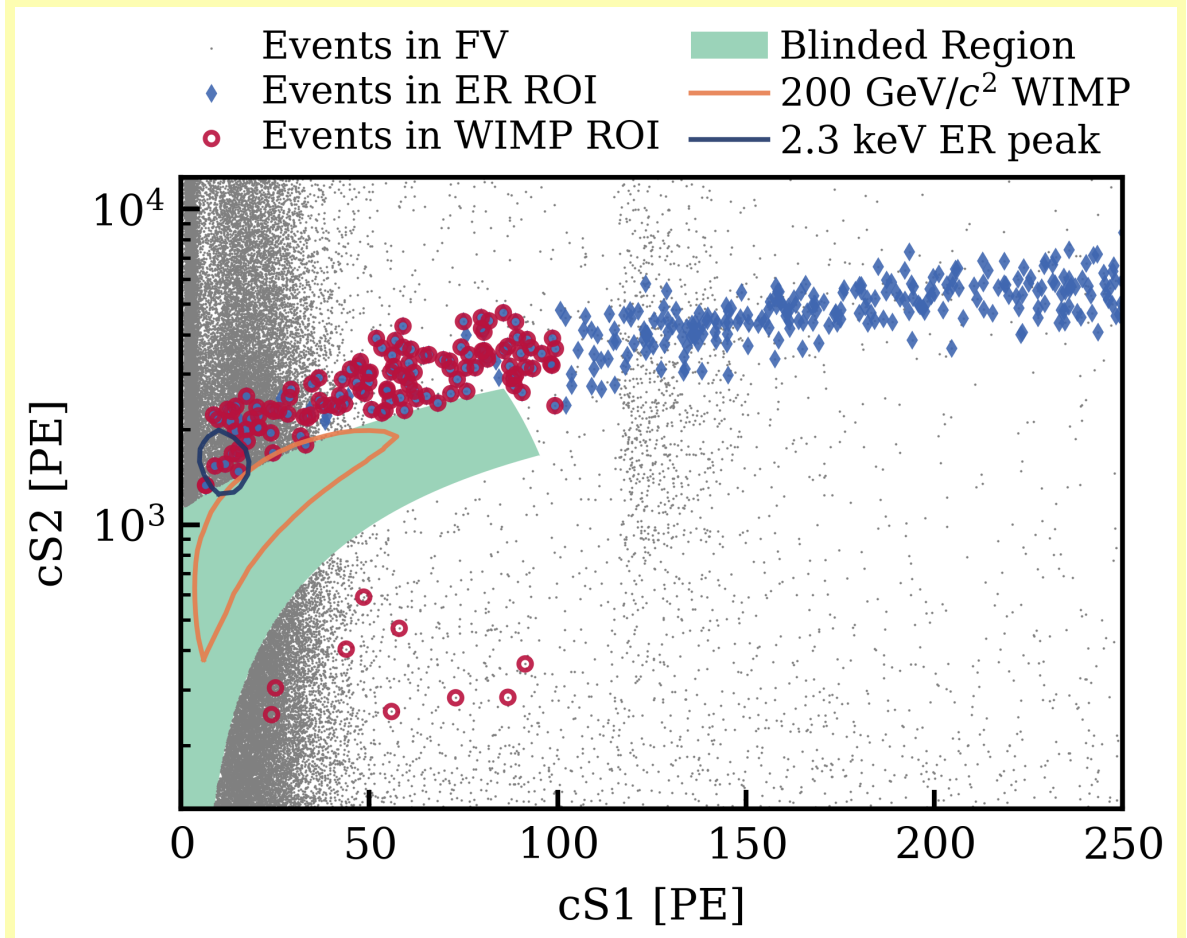


EXPERIMENTER BIAS IS A DANGER WITH FEW EVENTS

- The most common experimenter bias mitigation method is “blinding” — not showing the signal-like region of parameter space until the analysis has been frozen
- LUX developed a “salting” procedure where synthetic signals were made by stitching together genuine S1 and S2 signals into full events and placing them in the

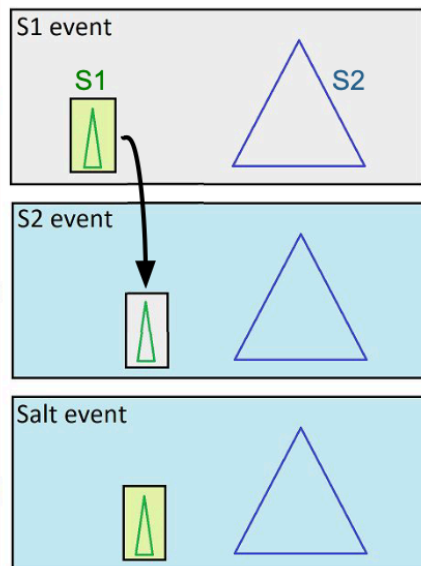


Tyler Anderson “Salting as a Bias Mitigation Technique in LZ”, presentation at LIDINE 2021

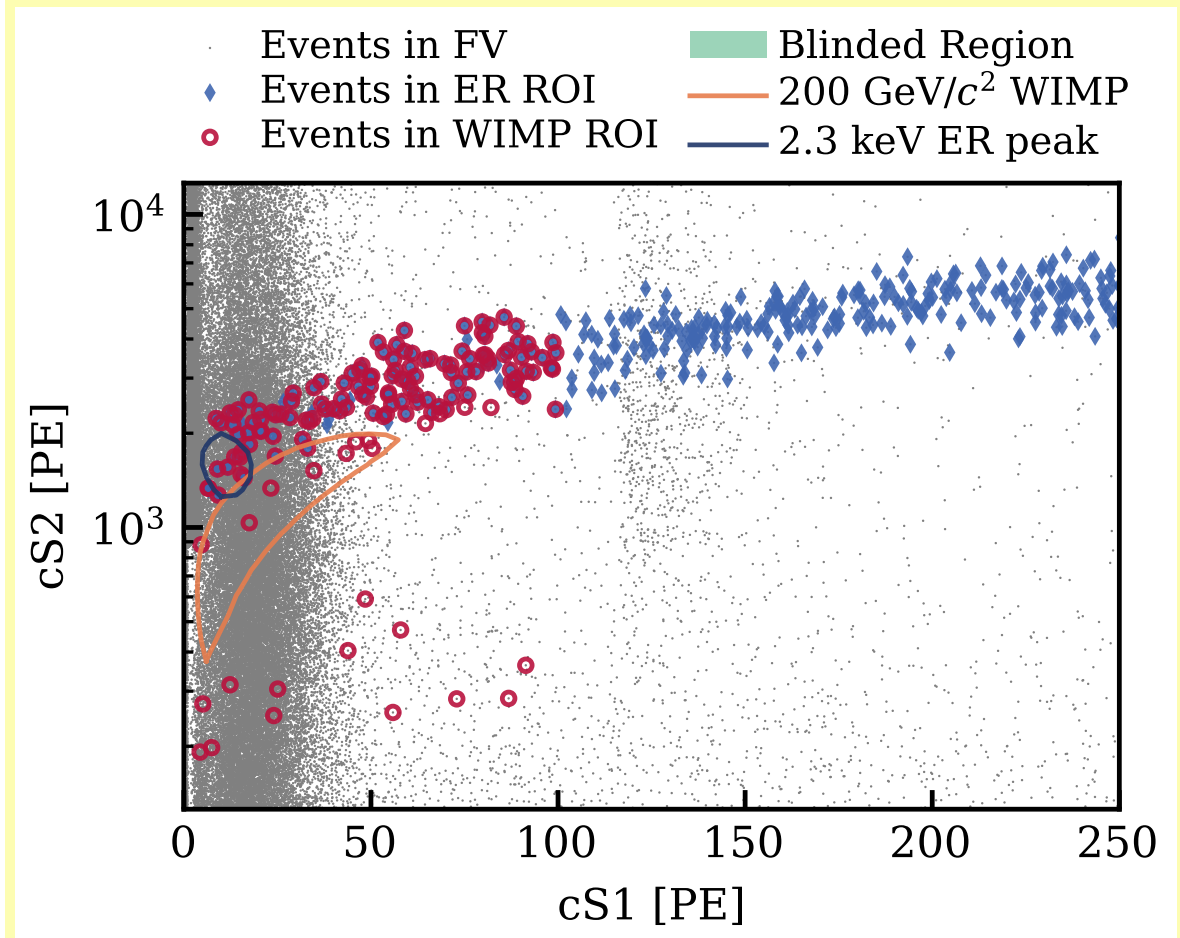


EXPERIMENTER BIAS IS A DANGER WITH FEW EVENTS

- The most common experimenter bias mitigation method is “blinding” — not showing the signal-like region of parameter space until the analysis has been frozen
- LUX developed a “salting” procedure where synthetic signals were made by stitching together genuine S1 and S2 signals into full events and placing them in the

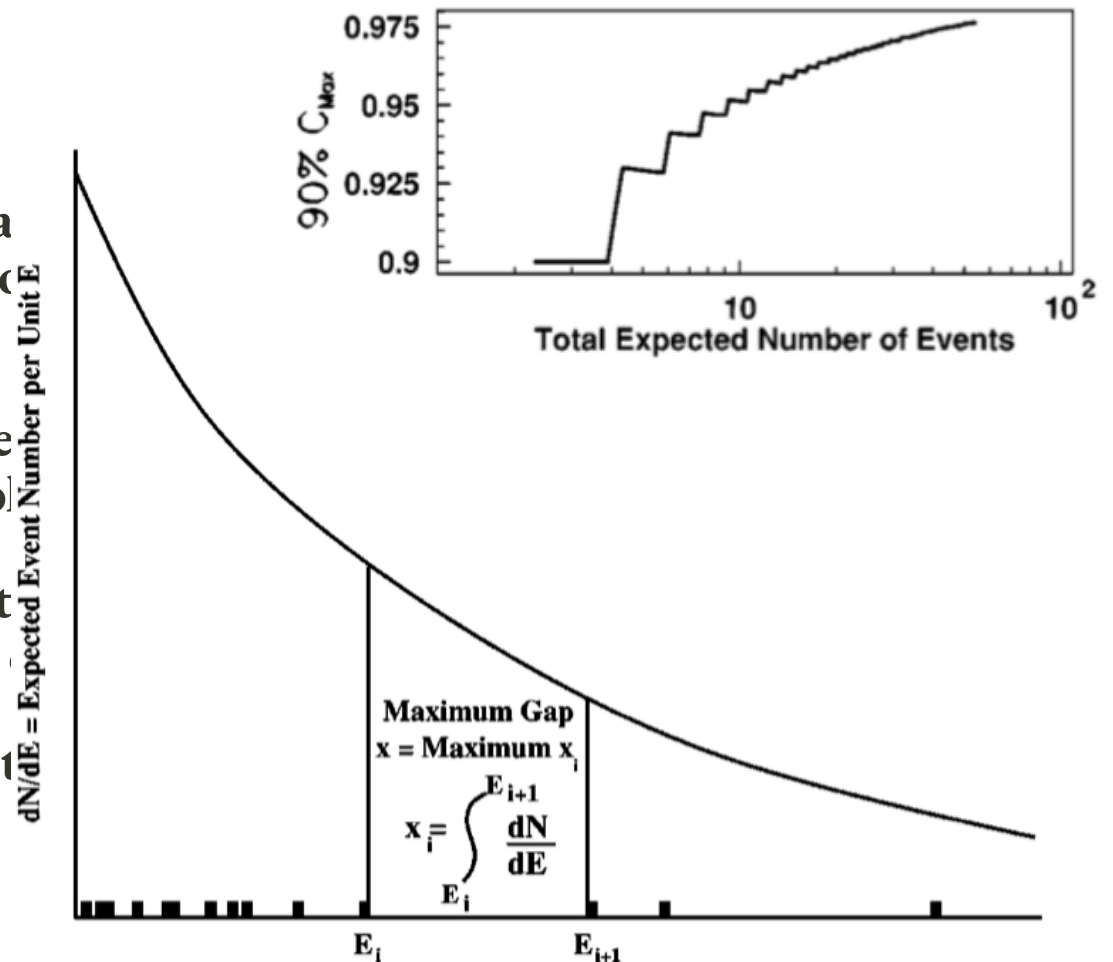


Tyler Anderson “Salting as a Bias Mitigation Technique in LZ”, presentation at LIDINE 2021



MAX GAP AND OPTIMUM INTERVAL

- If the signal distribution is known an optimal interval method can incorporate unknown background
- Find the space between observed events and find the largest signal compatible
- The method can be extended as “optimal largest interval containing 0,1,2 etc”
- threshold for the best interval test statistic



S. Yellin. Finding an upper limit in the presence of an unknown background. Physical Review D, 66(3), Aug 2002. ISSN 1089-4918. doi: 10.1103/physrevd.66.032005.

ALEA—CURRENT XENON TOOL

<https://github.com/XENONnT/alea/>

- Provides runners, submitters for toyMC profile construction
- And a flexible framework to add your own likelihood—define a functions for the likelihood and to generate data in a common form
- Includes a full simplified IXe TPC-style likelihood!
- Based on XENON1T and XENONnT SR0 WIMP analyses, with improved maintainable code

The screenshot shows the GitHub repository for 'alea'. The commit history table is as follows:

Commit Message	Author	Date
add more informative error msg (#229)	hammannr	2 days ago
Debug for pypl build (#197)	alea	last month
add more informative error msg (#229)	alea	2 days ago
Use pyproject.toml to install alea-inference (#192)	alea	last month
change defaults to also use .llhs in notebook	alea	10 months ago
Add file hash to bluaice hash (#225)	alea	2 days ago
Bump to v0.3.0 (#219)	alea	3 weeks ago
Rename submitters	alea	last year
Update hypotheses and canon_hypothesis by pre_proc...	alea	9 months ago
[pre-commit.ci] pre-commit autoupdate (#226)	alea	4 days ago
Use pyproject.toml to install alea-inference (#192)	alea	last month
Bump to v0.3.0 (#219)	alea	3 weeks ago
Initial commit	alea	last year
point away from alea for physics models (#143)	alea	8 months ago
Bump to v0.3.0 (#219)	alea	3 weeks ago
Use pyproject.toml to install alea-inference (#192)	alea	last month

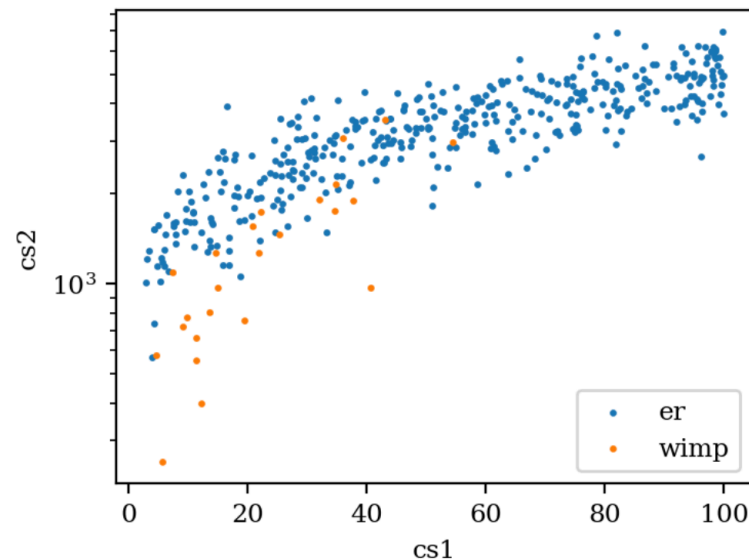
The README section contains the following text:

alea

alea is a flexible statistical inference framework. The Python package is designed for constructing, handling, and fitting statistical models, computing confidence intervals and conducting sensitivity studies. It is primarily developed for the [XENONnT dark matter experiment](#), but can be used for any statistical inference problem.

Alea aims to model the statistical behaviour of an experiment, which again depends on your knowledge of the underlying physics—this can range from the very simple, such as measuring a gaussian-distributed random variable, to complex likelihoods where each model component is created by physics simulations (GEANT4), fast detector simulations (for example [appletree](#) for XENONnT) or a data-driven method.

If you use alea in your research, please consider citing the software published on [zenodo](#).



Most WIMP events have lower values of $cs1$ and $cs2$ compared to the ER events. We can use this to discriminate WIMP signal events from ER background events.

HANDS-ON EXERCISE

The screenshot shows a GitHub repository page for 'direct_detection_ictp_2026' by user 'kdund'. The repository is public and has 1 branch and 0 tags. The commit history shows a commit by 'kdund' titled 'clear cells' from 16 hours ago, with 2 commits. The file list includes 'templates', 'README.md', and 'lxe_likelihood_example.ipynb'. The README file is selected, showing the repository name 'direct_detection_ictp_2026'.

File	Commit Message	Time
templates	repository init	16 hours ago
README.md	repository init	16 hours ago
lxe_likelihood_example.ipynb	clear cells	16 hours ago

**GITHUB.COM/KDUND/
DIRECT_DETECTION_ICTP_2026**